A Numerical Model of Deformation of Foliated Rock Slopes

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Abstract: Rock masses are often observed to be intersected by a single set of discontinuity such as foliation planes, bedding planes or joints giving rise to a layered (foliated) structure. The occurrence of such foliated rock masses is common in mining practices. If slopes are excavated in such rock masses, the individual rock layers may slide relative to each other, may separate (open up), may bend and fracture ultimately leading to slope failures. Methods employing continuum approximation in describing the deformation of such rock masses possess a clear advantage over explicit models. The introduction of moment (couple) stresses and internal rotations associated with the bending of the rock layers leads to a Cosserat-type theory. In the present model, the behaviour of the intact rock layer is assumed to be linearly elastic and the interfaces are assumed to be elastic perfectly plastic. Condition of slip at the interfaces are determined by a Mohr-Coulomb criterion with tension cut off at zero normal stress. The theory is valid for moderately large deformations. The model is incorporated into the finite element program AFENA and validated against an analytical solution of elementary buckling problems of a layered medium under gravity loading and a centrifuge experiment result.

KEY WORDS: Foliated Rock, Large Deformation, Buckling, Equivalent Continuum

1. Introduction

Often, rock masses are observed to be intersected by a single set of steeply dipping discontinuities, such as regular bedding planes, foliation or joints, producing so called foliated (layered) rock masses. Foliated rock mass is common in mining practices. When a slope is excavated in such a rock mass, besides the normal slip type of failure, two additional modes of failure have to be considered: 1) if the discontinuity plane dips inside the rock mass (Figure 1a), the individual rock layers may bend into the excavation thus giving rise to a class of so called flexural toppling failures; 2) when the discontinuity plane dips more or less parallel to the slope face (Figure 1b), the rock layers may buckle under their own weight giving rise to a buckling mode of failure. In figure 1, tentative outlines of the slopes after deformation are sketched by dashed lines.

Toppling failure of foliated slopes are observed quite frequently. For example, statistics presented by Swindells (1990) showed that in 1989, 44% of all open-pit gold mines in Western Australia suffered major wall failures in which toppling failure of hangingwall were the predominant failure mechanism in a random sample of 63 open
pits. Toppling failures occurred in foliated rocks of varying lithology and varying joint spacing ranging from a few centimeters to approximately 0.8 m.

![Figure 1. A schematic of slopes in a foliated rock mass](image)

(a) Flexural toppling and (b) Flexural buckling

There are essentially two different approaches that can be adopted in simulating the behaviour of foliated rock masses. Firstly, the foliated rock mass can be described in a discontinuum manner, such that each and every joint and intact rock layer are discretely defined in numerical simulation (Goodman et. al, 1968). However, when closely spaced joints occur in large numbers such that the layer thickness becomes much smaller than the relevant structural dimensions, the discrete modelling of such a medium becomes tedious and expensive to perform. The only feasible and effective way of incorporating the influence of such a system of joints in the analysis is to formulate an equivalent model within the framework of continuum mechanics with appropriate constitutive relationship.

Such a continuum description of layered materials can be formulated on the basis of the Cosserat theory (E & F Cosserat, 1909). The Cosserat theory of elasticity incorporates a local rotation of material points in addition to the translation assumed in the classical continuum, and couple (moment) stresses (moments per unit area) in addition to the classical force stresses (forces per unit area). In Muthiaaus (1993) and Zvolinski and Shkhinek (1984), a Cosserat model was formulated for the case of elastic joints. A Cosserat model with elastic-plastic joints was formulated in Adhikary et. al (1996), Adhikary and Dyakin (1997). In these earlier models, the deformation is assumed to be infinitesimal and hence their applicability is limited.

The purpose of this paper is to present a model that is also valid for moderately large deformations and to apply this model for analysing flexural toppling and flexural buckling phenomena in foliated rock masses. We restrict ourselves to "moderately" large deformations for two reasons: firstly, the assumption is perfectly valid in the buckling type problems considered here and secondly, the implementation requires only minor modifications to the existing Cosserat based numerical code initially designed for infinitesimal deformations. This paper briefly describes the theory and illustrates its potential by means of examples.
2. Cosserat theory

a) Cosserat deformation measures in plane strain

We consider infinitesimal deformations first and then subsequently propose a simple generalisation for moderately large deformation. A local rigid cross is assigned to every material point \((x, z)\) of the body in a Cartesian coordinate system \((X, Z)\). The layering is assumed to be parallel to the \(X\)-direction. In the process of deformation, the material points (rigid crosses) may undergo rotations by an angle \(\Omega_y\) about the \(y\)-axis in addition to the translations given by the displacements \((u, w)\).

For the formulation of constitutive relationships, we need deformation measures which are invariant with respect to rigid body motions. In the conventional, infinitesimal continuum theory, this requirement leads to the strain tensor,

\[
\varepsilon_y = \frac{1}{2} \left( u_{i,i} + u_{i,j} \right), \quad u_{i,j} = \frac{\partial u_i}{\partial x_j}, \quad u_1 = u, \quad u_2 = w, \quad x_1 = x, \quad \text{and} \quad x_2 = z
\]

(1)

In the Cosserat theory, we have two other invariant deformation measures, namely

\[
\Omega_y^{rel} = \omega_y - \Omega_y, \quad \omega_y = \frac{1}{2} \left( u_{2,1} - u_{1,2} \right)
\]

(2)

which represents the relative rotation between the material element and the corresponding rigid coordinate cross, and

\[
\kappa_{yi} = \Omega_{yi}
\]

(3)

which is a measure for the relative rotation between neighbouring rigid crosses. It is usual to combine equations (1) and (2) into a single, tensorial deformation measure (Mühlhaus, 1993), namely:

\[
\gamma_{xx} = \partial u / \partial x, \quad \gamma_{xz} = \partial u / \partial z + \Omega_y, \quad \gamma_{zz} = \partial w / \partial z, \quad \gamma_{xx} = \partial w / \partial x - \Omega_y
\]

(4)

Then,

\[
\varepsilon_y = \frac{1}{2} \left( \gamma_{yy} + \gamma_{ji} \right), \quad \Omega_y^{rel} = \frac{1}{2} \left( \gamma_{xx} + \gamma_{zz} \right)
\]

(5)

For the treatment of large deformation problems, we require an appropriate generalisations of the relative deformation tensor \(\gamma_{ij}\) and the curvature tensor \(\kappa_{ji}\) (here, \(i, j = x, z\)). Perhaps the most straightforward generalisation of \(\gamma_{ij}\) reads (Mühlhaus, 1995):

\[
\Gamma = \left( R^e \right) F
\]

(6)

where, \(F\) is the deformation gradient and \(R^e\) is the rotation tensor of the Cosserat rigid cross. When the Cosserat rotation is equal to the rotation \(R\) of an infinitesimal element of the continuum, then it follows from the polar decomposition \(F = RU\) such that
\[ \Gamma = U, \] where \( U \) is the right Cauchy-Green tensor (Mühlhaus, 1995). For infinitesimal deformation \( \Gamma \), reduces to \( \gamma \).

The starting point for the derivation of large deformation theory is the virtual work principle. This paper deals with only a relatively simple variant of the general theory similar to that assumed in the engineering beam and plate buckling theories in which the geometric non-linearity associated with the curvatures and couple stresses is neglected. In the incremental form of the virtual work expression, this can be expressed as (Mühlhaus, 1995):

\[ \Delta \delta W = \int_b \Delta \sigma_y \delta y dV + \int_b \sigma_y \Delta \delta \gamma_y dV + \int_b \Delta m_{yj} \delta \kappa_{yj} dV \]

where

\[ \Delta \delta \gamma_y = \delta \Omega_{ki} \Delta \gamma_{yj} + \Delta \Omega_{ki} \delta \kappa_{yj} \quad \text{and} \quad \Omega_{ki} = -\varepsilon_{i,0} \Omega_y \]

Here, \( \varepsilon_{i,j} \) is the permutation symbol, \( b \) denotes the domain of integration in the undeformed configuration. Terms of third and higher order have been dropped. It should be mentioned that in the present formulation \( \sigma_y \) does not designate the Cauchy stress tensor, but rather the energy conjugate to \( \Gamma_y \).

From the principle of virtual work, a simple calculation yields the following system of equations:

\[ t_{ij,j} = 0 \quad \text{and} \quad \varepsilon_{ij} \left( \Delta \tau_{ij} - \sigma_{jk} \Delta \gamma_{jk} \right) = 0 \]

where,

\[ t_{ij} = \Delta \tau_{ij} + \sigma_{ky} \Delta \Omega_{ki} \quad \text{and} \quad \Delta \tau_{ij} = \Delta \sigma_{ij} - \Delta m_{ij} \]

By an expansion of (9) with the use of (10), the local moment and angular moment balance are obtained as:

\[ \Delta \sigma_{xx} - \left( \sigma_{xx} \Delta \Omega_y \right)_x + \Delta \sigma_{zz} - \left( \sigma_{zz} \Delta \Omega_y \right)_z = 0 \]

\[ \Delta \sigma_{xx} + \left( \sigma_{xx} \Delta \Omega_y \right)_x + \Delta \sigma_{zz} + \left( \sigma_{zz} \Delta \Omega_y \right)_z = 0 \]

\[ \Delta \sigma_{xx} - \Delta \sigma_{zz} - \Delta m_{xx} \sigma_{xx} - \sigma_{xx} \Delta \gamma_{xx} + \sigma_{xx} \Delta \gamma_{xx} - \sigma_{zz} \Delta \gamma_{zz} + \sigma_{zz} \Delta \gamma_{zz} = 0 \]

b) Constitutive relationships

The layer interfaces can exhibit three different modes of behaviour: (a) elastically connected with the interface normal and shear stiffness \( (k_n \) and \( k_s \), (b) disconnected with frictional sliding and (c) disconnected with tensile opening. This kind of behaviour is best described by relations of elasto-plastic type. With this in mind, the rate of the deformation vector are decomposed into an elastic and a plastic part (see Eq. 17 and 18 for the definition of pseudo vectors), \( \text{viz} \).
\[ \dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_p \]  

(14)

For the criterion of interface sliding, \( f_s \), and the corresponding plastic potential, \( g_n \), we assume:

\[ f_s = |\sigma_{zz}| - tg \phi_j \sigma_{zz} - c_j \leq 0, \quad g_s = |\sigma_{zz}| \]  

(15)

where \( \phi_j \) and \( c_j \) designate the angle of friction and the cohesion of the joints respectively. Similarly, the criterion for the tensile opening and the corresponding plastic potential are written as:

\[ f_t = \sigma_{zz} \geq 0, \quad g_t = \sigma_{zz} \]  

(16)

For the explicit representation of the stress-strain relationship in terms of tensor components, it is advantageous to introduce pseudo-vector and matrix notation as follows:

\[ \bar{\sigma} = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \mu_{xy}, \mu_{xz} \} \]  

(17)

\[ \bar{\varepsilon} = \{\gamma_{xx}, \gamma_{yy}, \gamma_{zz}, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}, K_x, K_y, K_z \} \]  

(18)

The superscribed arrows distinguish pseudo-vectors from true vectors and tensors. The elasto-plastic relationships in the general form can be expressed as:

\[ \dot{\varepsilon} = [D_{ep}] \dot{\varepsilon} \]  

(19)

where

\[ D_{ep} = D_e - \alpha \frac{D_e \begin{bmatrix} \frac{\partial \bar{\varepsilon}}{\partial \bar{\sigma}} \\ \bar{\sigma} \end{bmatrix}^T D_e}{\begin{bmatrix} \frac{\partial \sigma}{\partial \sigma} \\ D_e \begin{bmatrix} \frac{\partial \sigma}{\partial \sigma} \end{bmatrix} \end{bmatrix}} \]  

(20)

Here \( \alpha \) is defined as:

\[ \alpha = 1 \quad \text{if} \quad f_s = 0 \quad \text{and} \quad \lambda (\text{plastic multiplier}) > 0; \quad 0 \quad \text{if} \quad f_s < 0 \quad \text{and/or} \quad \dot{\lambda} \leq 0 \]  

(21)

The course of derivation leading to Eq. (20) is exactly the same as in standard continua so we need not to go into further details here. For the present layered material, the elasticity matrix \([D_e]\) reads

\[ D_e = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & 0 \\ G_{11} & G_{12} & 0 & 0 \\ G_{21} & G_{22} & 0 & 0 \\ \text{symm} & \text{symm} & B & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  

(22)
where $A_{11}$, $A_{12}$ and $A_{22}$ are the conventional anisotropic elastic moduli and $G_{11}$, $G_{12}$, $G_{22}$ and $B_f$ are the new Cosserat moduli which can be expressed as:

\[
A_{11} = \frac{E}{1 - \nu^2 - \frac{\nu(1+\nu)^2}{E}}, \quad A_{22} = \frac{1}{1 - \nu^2 + \frac{E}{hk}}, \quad A_{12} = \frac{\nu}{1 - \nu} A_{22}
\]  

(23)

\[
\frac{1}{G_{11}} = \frac{1}{G} + \frac{1}{hk}, \quad G_{11} = G_{12} = G_{22} = G_{11} + G
\]  

(24)

and

\[
B_f = \frac{E h^2}{12(1 - \nu^2)} \frac{(G - G_{11})}{G + G_{11}}
\]  

(25)

where $E$ is the Young’s modulus, $\nu$ is the Poisson’s ratio, $h$ is the thickness and $G$ is the shear modulus of the intact layer.

3. Numerical analysis

a) Numerical verification

The large deformation model has been implemented into the finite element code AFENA (Carter and Balaam, 1995). The finite element formulation is described in detail in Adhikary et. al (1997). For the verification of the model, the critical selfweight for a package of vertical layers was computed and compared with the analytical solution. It was assumed that all the layers in the package buckle in the same manner.

Figure 2. (a) Finite element mesh and the boundary conditions, (b) Deformed mesh

Figure 2a presents the finite element mesh and the prescribed boundary conditions adopted in this study. The problem region was discretised into 200 8-noded isoparametric quadrilaterals. The height ($H$) of the package composed of fifteen 1 m thick ($h$) layers was 30 m, the Young’s modulus ($E$) was assumed to be unity and the Poisson’s ratio ($\nu$) was taken to be 0.16. The layer interface (joints) were assumed to
possess zero shear stiffness and a normal stiffness \( k_n h \gg E \). A small perturbation (i.e. a small horizontal load, \( f_h \)) was applied at the top corner of the model.

![Graph showing load deflection plot for the case of a package of vertical layers.](image)

Figure 3. Load deflection plot for the case of a package of vertical layers

The problem was solved incrementally by increasing the self-weight (\( \gamma \)) of the layers in equal steps of \( 10^6 \) kN/m\(^3\). Equilibrium iterations were conducted within each increment. In this study, a Newton-Raphson iteration scheme was adopted, where the displacement convergence tolerance (Bathe, 1982) is set at \( 10^{-6} \). Figure 2b presents the deformed mesh as the applied load approached the critical value.

Figure 3 presents the load-deflection curve obtained from the numerical computation. The analytical result, shown in this plot by a dashed line, was computed using the following expressions (Timoshenko, 1956):

\[
\frac{\gamma H^3}{E h^2} = 0.6575(1 - v^2)^{-1}
\]

(26)

It can be seen clearly that the computed critical load is in agreement with the theoretical solution (Eq. 26).

b) A rock mechanics application

**Flexural buckling of slopes** - The numerical code is further used to model an example of a buckling problem in a foliated rock mass. The slope geometry and the finite element mesh used in this analysis is shown in Figure 4. The rock layers were assumed to dip parallel to the slope face at an angle of 80°. The layers are assumed to be 1 m thick and possess the same elastic parameters as adopted in the previous example. The problem region was discretised into 410 8-noded isoparametric quadrilaterals. The numerical calculations were conducted for the case of three different joint friction values, i.e. 0°, 10°, 30°.

Figure 5a presents the horizontal displacements of the node (shown by an arrow in Figure 4) for the case of zero joint friction. Results of both large deformation and small deformation analysis are shown in the figure. It can be seen that the small deformation model substantially under-estimates the deformation. However, the effect of large deformation becomes important only at higher load levels. The discrepancies
between two model results are marginal at the load levels lower than about 50% of the critical buckling load.

Figure 4. Finite element mesh and the boundary conditions used in the buckling analysis

The dependency of a representative horizontal displacement on the loading parameter is represented in Figure 5. The results show clearly, how a higher joint friction tends to increase the effective strength of the layered medium as a whole resulting in a higher buckling load. Figure 6 presents the deformed mesh as the applied load approached the critical value.

As compared to the previous result, the critical non-dimensional buckling parameter is about 40 times higher for the case of zero joint friction angle and 70 times higher for the joint friction angle of 10°. This increase in the critical buckling load is firstly due to the fact that the buckling modes observed are different and secondly in the case of the slope example, the self weight of the rock, will always impart a component of compressive stress to the individual rock layer in the direction perpendicular to the layering. Hence, this compressive stress will resist to a certain extent the buckling stresses giving rise to the higher critical loads.

Figure 5. Load deflection curve for the case of buckling slope
Figure 6. Deformed mesh as the applied load approached the critical value (displacement magnification = 10)

**Flexural toppling of slopes** - The mechanism of flexural toppling of slopes has been investigated in detail in (Adhikary et. al 1996 and 1997). Here, we restrict ourselves to a brief outline of the most important results.

![Comparison of experimental and numerical results.](image)

Figure 7. Comparison of experimental and numerical results.

The finite element mesh was similar to the one used in the previous example. Figure 7, presents the geometry, mechanical properties and the comparison of the horizontal displacements measured during the centrifuge tests and those computed using the numerical code. A cohesion of 16.5 kPa was used as a matching parameter in the numerical analysis. The computed and the experimental displacements can be seen to be in a remarkably good agreement.

4. **Conclusions**

In this study, a large deformation model of foliated rock masses with elastic layers of equal mechanical properties and equal thickness has been devised on the basis of the Cosserat continuum theory and implemented into a finite element code. The layer interfaces (joints) are assumed to be rigid-perfectly plastic and are allowed to open in tension and yield in shear. It is found that the model can accurately predict the buckling of a package of layers subjected to gravity loading. It is shown, through an example of an excavation constructed in a layered rock mass, that small deformation
model may substantially under-estimate the deformation resulting in an over-estimation of the stability of the slope. However, in the considered example, the effect of large deformation becomes important only at higher load levels. The discrepancies between two model results are marginal at the load levels lower than about 50% of the critical load.

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6 References


