Numerical Modelling of Hydraulic Fracturing in a Porous Medium

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Abstract: Hydraulic fracturing is a technique used in many engineering applications. Most fracturing analyses are based on fracture mechanics for a cohesive and impervious material using the stress intensity factor and fracture toughness within the framework of linear elastic fracture mechanics. The fracturing of a porous material requires the analysis of pore fluid diffusion and leak off, which have a significant effect on the fracturing of the material. Also, the temperature of the injecting fluid and the ground can be quite different leading to thermal expansion or contraction and shearing of the material. Numerical treatment of hydraulic fracturing in a porous material is complex due to the interaction of heat, pore fluid and mechanical effects of material. In this paper, a numerical scheme for modelling hydraulic fracturing is presented.

In modelling fractures, the discrete fracture approach is adopted where fractures are penetrating the finite element domain along element boundaries. Leak off of fluids and exchange of heat are fully coupled with mechanical deformation of the material. It is found that depending on the cohesive strength of the material and the permeability of the material relative to the injection rate, different fracture mechanisms which are quite different than those considered in conventional fracture analysis emerge. This is due to the reduction of effective stresses at the fracture tip which leads to shear fractures as well as tensile parting. Fracture initiation and propagation of the material can be modelled effectively using the proposed finite element scheme. The application of the model in a porous material is illustrated with examples.

Key Words: Hydraulic fracturing, fracture analysis, finite element method, in-situ oil recovery.

Introduction

Hydraulic fracturing refers to the parting of geomaterials due to excessively high fluid pressure. Hydraulic fracturing is both a natural phenomenon as well as a technique used by engineers to alter the in-situ ground condition. For example, in the case of an earth dam, impounding of the reservoir behind the dam increases the pore pressure in the dam and its foundation. The increase in pore water pressure can be sufficiently high to cause the foundation and the dam to fracture. Engineers also make use of hydraulic fracturing to change the flow characteristics of the ground. For example, in recovering oil in an oil rich deposit, high temperature steam is often used to reduce the viscosity of the bitumen which is embedded in the sand matrix. Heating by conduction usually takes a considerable amount of time causing delay in oil production and increase in production cost. By fracturing the ground, steam can travel through open
cracks thus speeding up the heating process considerably. Therefore hydraulic fracturing is beneficial when it is properly used.

Hydraulic fracturing has made a significant contribution to enhancing oil and gas production rates. Since its inception, hydraulic fracturing has been developed from a simple, low-volume, low rate reservoir stimulation technique to a highly engineered and complex procedure that is used for many purposes. Figure (1-1) depicts a typical hydraulic fracturing process in the petroleum industry. The procedure is as follows: first, a fluid such as water (called 'pad') is pumped into the well at the desired depth (pay zone) to initiate the fracture and to establish its propagation. This is followed by pumping a slurry of fluid mixed with a propping agent such as sand (which is often called a 'proppant'). This slurry continues to extend the fracture and concurrently carries the proppant deeply into the fracture. After pumping, the injected fluid chemically breaks down to a lower viscosity and flows back out of the well, leaving a highly conductive propped fracture for oil and gas to flow easily from the extremities of the formation into the well. It is generally assumed that the induced fracture has two wings extending in opposite directions from the well and is oriented more or less in a vertical plane. Other fracture configurations such as horizontal fractures are also reported to occur, but they constitute a relatively low percentage of the situations documented. Experiences indicate that at depths below 600 meters (2000 ft), fractures are usually oriented vertically. At shallow depths, horizontal fractures have been reported (Veatch Jr. et al., 1989). The fracture pattern, however, may not be the same for different types of soils and rocks.

Over the years, the technology associated with fracturing has improved significantly. A number of fracturing fluids (injactant) have been developed for different types of reservoirs ranging from shallow, low-temperature formations to those located in deep, hot areas. Many different types of proppants have been developed, ranging from silica-sand (standard) to high-strength materials, such as sintered bauxite. The latter is used in the deep formations where fracture closure stresses exceed the sand capabilities. New analytical and diagnostic methods and design models have emerged, and the service industry has continually developed new equipment to meet emerging challenges.

Although technology in hydraulic fracturing is advancing significantly, its design still involves a good deal of judgment and practical experience. After 50 years of fracturing practice and research, the ability to determine the fracture shape, dimensions (length, width, height), azimuths, and fracture conductivities is still not fully developed. In addition, the ability to measure in-situ rock properties and stress fields which have a significant effect on fracture propagation is not perfected. Consequently, the optimization of hydraulic fracturing treatments is often subject to limitations. A numerical model capable of analyzing different aspects of reservoir engineering as well as fracture mechanics can be a valuable tool to overcome many uncertainties in the design of hydraulic fracturing and help the industry to optimize the process. As a result, reservoir engineering and fracture mechanics are two important subjects that should be used for developing the numerical model.

**Petroleum Reservoir Simulation**

Simulation of petroleum reservoirs requires a clear understanding of flow in porous media. Since oil, water, and gas can flow simultaneously in a heavy oil reservoir, it is
necessary to understand the mechanism of multiphase unsaturated flow. The effect of high temperature, which causes interactions between oil, water, and gas, further complicates the problem. In general, the pressures of these three phases can be different and the pressure difference between any two phases is attributed to 'capillary pressure'. Large changes in pressures either cause gas to dissolve into the fluid or the fluid to transform into the gas phase. This results in a new mass balance in the system which causes the degree of saturation of each phase to change.

For decades petroleum engineers have been developing numerical simulators for modeling oil reservoirs and hydraulic fracturing. These simulators solve the fluid mass balance equation and/or heat transfer equation in the reservoir. Most of these simulators are based on the finite difference method and are developed for some special boundary conditions. The degree of sophistication of these simulators varies considerably. In some cases, the ability to make a reasonably accurate prediction of the response of a reservoir is poor. This is due, in part, to the lack of geomechanics in some reservoir models (Tortike, 1991). A detailed deformation analysis of the reservoir is required to improve the results of the conventional reservoir simulators.

Modeling of a thermal multiphase flow in a deformable oil reservoir requires coupling of at least three basic conservation laws for fluid flow, heat flow and applied loads. Where uncemented heavy oil deposits such as oilsands are of interest, the peculiar characteristics of oilsand should also be considered in the model. Oilsand exhibits significantly different behavior from that of typical cemented sandstone and limestone (which are usually characterized by linear and nonlinear elastic behavior, respectively). Oilsand's behavior which will be discussed later is elastoplastic and temperature dependent.

**Basic Formulation for a Coupled Thermal Flow Mechanical Analysis**

There are basically three sets of equations which must be satisfied in a coupled analysis. These equations are equilibrium equation, continuity equation for flow and energy transfer equation for thermal consideration. The equilibrium equation for a continuum can be written as:

\[
\sigma_{ij} + F_i = m'V_i + c'V_i,
\]

(1)

where \( \sigma_{ij} \) is stress tensor at any point

\( F_i \) is external load

\( V_i \) is soil matrix velocity

\( m' \) is mass coefficient

\( c' \) is damping coefficient

\( i, j \) are indices taking 1, 2 and 3 representing coordinate axes.

Normally in numerical implementation, the equation is expressed in incremental integral form as:

\[
- \int_A \sigma_{ij} \delta_{ij} \, dV + \int_A P \delta_{ij} \, dV = -\int_S \sigma_{ij} \omega \, ds + \int_A (-\Delta F_i + m' \Delta U_i + c' \Delta U_i) \omega \, dV
\]

(2)

where \( P \) is the pore pressure

\( \delta_{ij} \) is Kronecker delta

\( \omega \) is weighting function
S_c  portion of surface subjected to traction
V    volume of material.

When considering fluid flow in porous medium, the continuity equation is given by:
\[ \nabla (\rho v) - G = -\frac{\partial}{\partial t} (\rho\phi) \]  \( (3) \)
where \( \rho \) density of fluid
v velocity vector of flowing fluid
G fluid mass flux from sink (output) or source (input)
\( \phi \) porosity of soil mass
t time and
\( \nabla \) gradient operator \((\partial/\partial x_i)\)

which can be expressed in integral form as:
\[ \int_V (\rho \phi + \rho \phi) \omega dV \, - \int_V (\rho v) \omega dV \, = \int_S (\rho v) n \omega dS \]  \( (4) \)
where \( n \) is the normal to the surface on the boundary.

The governing equation of heat transfer is:
\[ \nabla \cdot (I_e) - Q = -\frac{\partial (\rho E)}{\partial t} \]  \( (5) \)
where \( I_e \) volumetric thermal energy flux
E internal energy per unit mass
Q energy flux from source (input) or sink (output)

Again the integral form of the equation can be written as:
\[ \int_S (I_e n_i) \omega dS \, - \int_V L_e \omega dV \, + \int_V (\rho E) \omega dV = 0 \]  \( (6) \)

Equations 2, 4 and 6 must be solved simultaneously. By making the usual
discretization of the domain in terms of nodal displacements, pore pressure and
temperature, the matrix equation for coupling all three process can be written as:
\[
\begin{bmatrix}
K_{11} & K_{12} & K_{13} \\
K_{21} & K_{22} & K_{23} \\
K_{31} & K_{32} & K_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta U^* \\
\Delta P^* \\
\Delta T^*
\end{bmatrix} = \begin{bmatrix}
F_1 \\
F_2 \\
F_3
\end{bmatrix}
\]  \( (7) \)
where \( K_{ij} \) are matrices based on equations 2, 4 and 6 and finite element
discretization of the domain
\( \Delta U^*, \Delta P^*, \Delta T^* \) are the incremental nodal displacements, pore pressure and
temperature vectors respectively,
\( F_1, F_2, F_3 \) are combined nodal forces, fluid and energy flux vectors.

**Hydraulic Fracture Modeling**

In numerical analysis, fracture can be modelled by three different approaches, the
discrete approach, the smeared approach and the dual porosity approach. Discrete
fracture approach is used where few fractures exist. In contrast, when the fracturing is so intense that the whole medium can be represented by a uniformly damaged material with modified material characteristics (for example modified equivalent stiffness and/or permeability), the smeared approach would be a more reasonable choice. It should be noted that in the smeared approach no fracture is introduced inside the medium and fractures are modeled by modifying the material characteristics in the fractured zone. Therefore the basic assumptions of continuum mechanics hold true for the smeared approach but not for the discrete approach. This is illustrated in Figure (2). The dual porosity approach is basically used for ‘naturally fractured’ oil reservoirs. These types of reservoirs in theory, are modeled as blocks stacked over each other with low porosity and low permeability. Between the blocks, fractures with very high porosity and permeability exist. Fluid flow in the reservoir consists of two parts: flow through the fractures with high permeability and flow through the blocks with low permeability.

Fracture mechanics theories, which were originally developed for metals, have been used successfully for geological materials in recent years. Linear elastic fracture mechanics (LEFM) or elasto-plastic fracture mechanics (EPM) have been used for analyzing fractures in soils and rocks. They are used to establish criteria for crack initiation, crack propagation, and crack arrest (which will be discussed later). Settari and Raisbeck (1979) investigated the result of combining a planar fracture model and a single phase compressible fluid flow model assuming elastic behaviour. Settari (1980) and Settari and Raisbeck (1981) further developed their previous work to include two phase thermal fluid flow. Modeling of fracture requires a knowledge of geological conditions in the ground. Local stress fields and variations of stresses between adjacent formations are often thought to be the main factors which control fracture orientation and fracture growth.

Regional stresses in the ground can have an impact on the azimuthal trend of the hydraulically created fractures. It is usually believed that the fracture propagates perpendicular to the direction of the minimum principal stress; i.e., tensile fracture is the prime mechanism in hydraulic fracturing. Recently, the possibility of shear failure before tensile failure has been of interest especially where the injection rate is high and the amount of fluid leak-off into the formation is significant.

The importance of performing a deformation analysis, once again emerges here because fracturing criteria are based on the stresses and deformations in the ground. Therefore, in order to obtain a realistic model for design purposes, the geomechanical behaviour of the ground has to be accounted for in reservoir simulation and hydraulic fracture analysis.

Mechanics of Fracture in Granular Material

When modeling fractures using the discrete fracture approach, it is necessary to first detect the occurrence of fracture. This is accomplished by examining the stress state in the entire domain to determine whether the fracture criteria are satisfied. Fracture can basically occur in three modes denoted by modes 1, 2 and 3 for tensile parting, in plane shearing and out-of-plane tearing respectively. For two-dimensional analysis, only modes 1 and 2 are applicable.

When considering fracturing of geomaterials, both modes 1 and 2 are possible and they can also occur simultaneously in one location. Unlike an impermeable material with
shear strength predominately derived from cohesion, fracturing in granular material can have different mechanisms. Consider a point in the ground where pore pressure is increasing. The increase in pore pressure results in a reduction of effective stress. Before any significant stress redistribution can occurs, the reduction in effective stress results in a decrease in mean stress as illustrated in Figure 3. Depending on the in-situ shear stress $q$ in the ground and the tensile strength of the material, this change in stress can follow one of the three stress paths shown in Figure 3.

For stress path 1, the initial shear stress is not high enough and the cohesion of the material is small which results in tensile parting of the material. Once the material has been fractured in tension, it is unlikely that it will be changed to a shear mode unless the fracture is closed and shear stress in increased locally. The orientation of fracture in this case is parallel to the major principal stress. Fracture in this case also results in breaking of the cohesive bonds of the material thus reducing the effective cohesion of the material to zero. Therefore unless the effective mean stress is increased, shear fracture cannot occur.

For stress path 2, the shear stress or the cohesive strength of the material is sufficiently high such that shear fracture occurs. In this case a localized shear band is formed where intensive shearing occurs. Also there will be an increase in permeability locally, which could lead to a decrease in pore pressure. If the permeability of the material is low and the fluid injection rate is high, there can be a local increase in pore pressure causing a decrease in effective mean stress. This is illustrated by path 3. In this case tensile fracture can occur after the material has been fractured in shear. Shear and tensile fracture directions are not the same.

Since pore fluid can penetrate the material ahead of existing fracture tip, both tensile and shear fractures can, in principal, occur ahead of an existing fracture tip. In hydraulic fracturing process, since fluid pressure is normally the highest near the injection zone, fracture is initiated in this zone and subsequent fractures are developed from existing fractures. Tensile fractures may not be ‘clean’ fractures in a granular material with zero cohesion since material in the fracture have been loosened and effective stress is basically zero, there many be high mobility of material in this zone resulting in a dramatically increase in local permeability.

In modelling fractures in a granular material, it is important to consider pore water diffusion and changes in effective stress state in the soil. The traditional criteria using linear elastic fracture mechanics without consideration of leakoff cannot model hydraulic fracturing in soil realistically.

**Numerical Techniques in Modelling Discrete Fractures**

Besides the development of a finite element model capable of coupling flow and deformation in modelling leakoff and changes in effective stresses, it is necessary to have a procedure of introducing discrete fractures in the model. In finite element analysis, the domain is subdivided into elements. Elements are joined together by enforcing nodal compatibility and equilibrium. When a fracture is formed, slip or separation will occur resulting in local incompatibility. However, equilibrium must still be satisfied. For tensile fracture, separation of elements will allow parting of material. Since zero boundary stress is a natural boundary condition for displacement finite element formulation, equilibrium is automatically satisfied when the material
formulation is based on effective stress. There are various ways of modelling discrete fracture in a finite element scheme (Ingraffea, 1977, 1984 and Ingraffea and Saouma 1984). Introduction of a fracture in a finite element mesh is accomplished by using multiple nodes and an effective numbering scheme.

The concept of multiple nodes is illustrated in Figure 4. For ordinary finite element design only one node is assigned for one location and all elements which are connected to that node will share the same node. However, in multiple finite element technique, each element has a separate node and the nodes are initially not connected even though they may have the same coordinate. Connection of the nodes is made either using Lagrangian multiplier technique or simply by proper referencing of the degrees of freedom of the nodes (Chan, 1981 and Pak, 1997).

When a fracture is formed the permeability along the fracture is increased locally. This is modelled by introducing a special fracture element capable of conducting fluid flow with no resistance to deformation. This is important since it allows increase in pore pressure at the new fracture tip without impeding deformation along the fracture.

**Numerical Example in Modelling Fracture**

The finite element mesh of an example problem illustrating a single fracture propagation is shown in Figure 5. It is anticipated that in this case fracture will be initiated at the left boundary where hot fluid is injected and propagates horizontally to the right boundary in the middle of the domain. Double nodes are used in the middle of the finite element mesh. Special fracture elements will be introduced when fracture occurs. A notch is created at the left side where high temperature fluid under high pressure is injected into the medium. The initial pore pressure and temperature in the medium are set to zero. A fluid flux of \(0.1 \times 10^{-5}\) m/sec and a heat flux of 10.0 J/sec are applied inside the notch. Generally, the induced stresses at the nodes are examined to determine whether the tensile or shear fracture criteria are satisfied. The criterion which is first satisfied governs the situation and causes the double nodes to split. The fracturing process continues until a static condition for the fracture is obtained. Material properties used in the analysis is shown in Table 1.

**Table 1: Summary of Material Properties used in the Analysis**

<table>
<thead>
<tr>
<th>Material Properties of Medium</th>
<th>Value</th>
<th>Material Properties of Fluid</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass coefficient (kN/m sec(^2))</td>
<td>0.00</td>
<td>Fluid thermal expansion (1/°C)</td>
<td>0.1 \times 10(^{-5})</td>
</tr>
<tr>
<td>Damping coefficient (kN/m sec(^{-1}))</td>
<td>0.00</td>
<td>Fluid compressibility (kPa(^{-1}))</td>
<td>0.5 \times 10(^{2})</td>
</tr>
<tr>
<td>Thermal expansion coeff. (1/°C)</td>
<td>0.9 \times 10(^6)</td>
<td>Heat capacitance (J/m(^2).°C)</td>
<td>0.00</td>
</tr>
<tr>
<td>Medium heat capacitance (J/m(^2). °C)</td>
<td>5.00</td>
<td>Fluid density (ton/m(^3))</td>
<td>1.00</td>
</tr>
<tr>
<td>Thermal conductivity (J/sec.m.°C)</td>
<td>20.0</td>
<td>Fluid viscosity (kPa.sec)</td>
<td>0.1 \times 10(^5)</td>
</tr>
<tr>
<td>Medium density (ton/m(^3))</td>
<td>0.00</td>
<td>Absolute permeability (m(^2))</td>
<td>5.5 \times 10(^{-12})</td>
</tr>
<tr>
<td>Modulus of elasticity (kPa)</td>
<td>0.6 \times 10(^8)</td>
<td>Gravity (m/sec(^2))</td>
<td>9.81</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial porosity</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 6 shows the propagation of fracture with time. Temperature and injection rate are kept constant with time. There is a 4 second delay before the initiation of fracture. Once fracture is initiated, it propagates fairly quickly to the right boundary. Figure (7) shows the variation of pore pressure at nodes 18 and 29 at the injection boundary and at nodes 1 and 45 which are far from the injection zone. It is seen that, the pore pressures are generally higher at the injection point. When 6 seconds have elapsed, fracture elements are activated. Since the permeability of the fracture elements are much higher than the medium, the pore pressure drops because the fluid suddenly finds easier paths to flow. After activation of all fracture elements the pore pressure again starts to increase. The effect of activation of fracture elements on nodes 1 and 45 (which are located far from the injection boundaries) is not large, as expected. The medium in general shows a monotonic increase in temperature with time.

Conclusion

Modelling discrete fracture is important in analyzing hydraulic fracture problems. In this paper, a coupled thermal, mechanical, flow, fracture model is introduced to analyze hydraulic fracturing for a porous material. Due to leak off and thermal expansion, fracture can be initiated ahead of existing crack tips. Different models of fractures has been examined and a numerical scheme for modelling discrete fracture is introduced. The proposed scheme is able to analyze discrete fractures without predetermined the location of fractures. Fluid flow in the fracture is also modelled using a special fracture element and leak off from newly formed fracture faces is also modelled. An illustrative example of modelling a single fracture propagation is provided.

Acknowledgement

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References


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**Figure 1:** Hydraulic Fracturing Process in Enhancing Oil Recovery  
(after Veatch Jr. et al. 1989)

**Figure 2:** Smeared and Discrete Crack Modelling

**Figure 3:** Stress Paths of Shear and Tensile Fracture Initiation of a Porous Material
Figure 4: Multiple Node Finite Element Technique in Fracture Modelling

Figure 5: Finite Element Mesh of Fracture Modelling Example

Figure 6: Propagation of a Single Fracture with Time

Figure 7: Pore Pressure Variation with Time Near and Far from Injection Zone