Some Recent Advances in the Modelling of Soft Rock Joints in Direct Shear

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Abstract: This paper describes some recent theoretical advances in the modelling of soft rock joints in direct shear. Careful observation of laboratory direct shear testing on rough concrete/rock joints has allowed purely theoretical models of behaviour to be developed. The processes modelled include asperity sliding, asperity shearing, post-peak behaviour, asperity deformation and distribution of stresses on the joint interface. Model predictions compare extremely well with laboratory test results. The models were also applied to direct shear tests on rock/rock joints. Although behaviour in general was well predicted, the strength of these joints were over-predicted. This paper briefly describes the testing carried out and the theoretical models developed. Reasons for the differences in behaviour of concrete/rock and rock/rock joints are presented.

Introduction

Rock joints are the planes of weakness in rock masses caused by past geologic processes. All rock bodies contain joints, which control the strength and stability of the rock mass. As demonstrated by the following partial list of applications, the behaviour of rock masses, and therefore rock joints, is crucial to the development of infrastructure and mining projects. Joints control the:

- design and construction of cut slopes for roads, railways and open cut mines;
- stability of rock abutments for dams;
- rock extraction strategies in underground mines;
- stability of natural rock slopes and the assessment of landslip potential;
- design of foundations for large structures on rock;
- viability of long-term waste storage solutions;
- design of support and lining solutions for tunnels and underground openings.

From the above wide-ranging list, it is obvious that rock joints have a significant influence on the activities of our community. The potential economic benefits to be gained by an improved model of joint shear behaviour (allowing steeper rock slopes, better design of support systems etc) are substantial. This paper describes some recent advances made at Monash University, in the development of a theoretical model for the behaviour of rock joints in direct shear.

Rock Joint Strength

Over the years, there have been many investigations into the shear behaviour of rock joints. Bandis (1990) groups these investigations as theoretical and empirical approaches. Both approaches have resulted in important contributions to our understanding of rock joint behaviour. The theoretical approaches (e.g. Hercogia, 1985; Saab and Amadei, 1990; Plesha, 1987; Carter and Ooi, 1988) usually utilise numerical methods (e.g. finite elements) coupled with advanced constitutive laws to model the behaviour of the joint interface. Although advanced mathematically, these constitutive laws are derived empirically from laboratory or field test results and do
not explicitly model the kinematics of shear development at the interface. As a result, they should only be used as predictive tools for situations that are properly represented by the database from which the empirical relationships were derived.

For this reason, many practical rock engineering solutions are based on “easy to use” empirical approaches. These approaches involve the analysis of test data to obtain correlations between joint and material characteristics and observed behaviour. The analyses may be particular to specific rock formations or individual projects, making them of limited general value, or they may offer more universal application, such as the JRC-JCS model developed by Barton and Bandis (1971, 1976, 1982, 1990). Since the mid 1970’s, the Barton and Bandis approach has dominated the practice of rock joint engineering and is now considered by many to be a de facto international standard. However, despite this high level of acceptance, there is concern regarding the highly empirical basis of the model. This concern often leads to more conservative design solutions.

This is not to say that empiricism per se is inappropriate for rock joint behaviour, but rather that it is more appropriate for local correlations than as the basis of a universal system. In order to develop a universal approach, a conceptual understanding of the physical processes and mechanisms, and theoretical models derived from such an understanding are necessary. Only in this way can realistic and reliable constitutive models be developed. The barrier to the advancement of such an approach to date has been the complexity of modelling natural and irregular systems such as rock joints. At best, the geometry of such joints can only be estimated statistically. Within the joint, the complex interacting processes of sliding, shearing, dilation, elastic and plastic deformation are all simultaneously at work. None of the theoretical or empirical approaches proposed so far have been predicated on the development of a fundamental understanding of these processes.

A Fundamental Approach to Concrete/Rock Joint Behaviour

The authors have conducted significant research on a problem with many parallels to the rock joint problem - that of the performance of bored piles in rock. The process of forming a concrete pile in rock creates a rough rock joint with a matching concrete, rather than rock face. As is briefly described in the following sections, the research has been based on a fundamental approach to modelling the joint kinematics.

The authors’ work is based on an extensive series of laboratory direct shear tests on concrete/soft rock joints. The soft rock used in the tests was an artificial siltstone called Johnstone (Johnston and Choi, 1986) with a uniaxial compressive strength of 3.5 MPa. The tests covered a range of roughness profiles and boundary conditions. Joint roughness was is one direction only and varied in complexity from simple idealisations of regular and irregular triangular asperities through to completely irregular and random profiles based on fractal geometry concepts (see Fig. 1).

The tests covered a wide range of boundary conditions appropriate to Civil Engineering piling applications rather than slope stability and mining applications, and were therefore conducted under constant normal stiffness (CNS) conditions rather than the more usual constant normal load (CNL) condition. In CNS tests the normal stress applied to the joint is directly proportional to the joint dilation, i.e. as the joint dilates, the normal stress applied to the joint is increased. The CNL condition is a subset of the CNS condition and corresponds to a normal stiffness of zero. Normal
subset of the CNS condition and corresponds to a normal stiffness of zero. Normal
stiffnesses ranged from \( K = 150 \) to \( K = 600 \) kPa/mm and initial normal stresses from
\( \sigma_n = 150 \) kPa to \( \sigma_n = 600 \) kPa. It should be emphasised that although global stresses
in the joints under these conditions may not be high in a geological context, local
contact stresses must reach the rock strength for asperity failure to occur. Tests were
conducted at a displacement rate of 0.5 mm/minute. Measurements of normal load,
shear load, dilation, and shear displacement were recorded by PC and displayed in
real time. Average shear stresses and normal stresses were automatically corrected
for area changes with shear displacement. All tests were recorded by video using time

![Regular Triangular](image1)

![Irregular Triangular](image2)

![Fractal](image3)

Figure 1 A selection of the roughness profiles used in direct shear testing

lapse photography. Full details of the testing are reported in Seidel (1993).
The purpose of testing simple regular triangular profiles was to obtain a basic
understanding of the kinematics of shear displacement. Such an understanding has
led to a better understanding of behaviour of the more realistic profiles.

The video records of tests on simple regular triangular profiles indicated that shear
behaviour involved two basic and apparently independent mechanisms; initial sliding
along the surface of the asperity (asperity sliding) and then shearing through the intact
asperity (asperity shearing). These two mechanisms have been reported previously by
several researchers including Patton (1966), Ladanyi and Archambault (1972) and
Lam and Johnston (1989). The video records of the tests on irregular triangular
asperity profiles showed that the shear behaviour of these more complex joints were
also controlled by these same two basic mechanisms. However, in the latter, there
was observed to be a complex interaction between sliding and shearing of individual
asperities. As the roughness profiles become more random and irregular, the
interaction between these two mechanisms become even more complex.

Shear displacement occurred initially by sliding up the steeper asperities. At some
stage the local stresses on the steepest asperity became too high and the asperity failed
in shear. Sliding then continued on the next steepest asperity until it too failed in
shear. This process continued with sliding occurring on progressively shallower
asperities. Based on these simple observations, the authors postulated that roughness
could be envisaged as a series of low angle long base length triangular asperities,
upon which were built successively higher angle and shorter base length asperities as shown in Fig. 2. The low angle, long base length asperities control behaviour at large shear displacements while the short steep asperities control behaviour at small shear displacements. The authors (Seidel and Haberfield, 1995a) further postulated that the distribution of asperity angles and base lengths could be described by a probability density function and a Gaussian distribution. Combining this simple concept with fractal geometry, the authors were able to develop a fractal model of roughness that could be used to generate realistic random rough joint profiles from simple regular triangular asperities.

Given that roughness can be generated in this way, a logical extension is that roughness can be abstractly modelled as a sequence of many individual triangular asperities of varying inclination and base length. That is, instead of generating roughness by compounding asperities on one another, they are placed side by side in series. As before, the distribution of asperity angles and base lengths is determined randomly from a Gaussian distribution. The behaviour of this abstract profile can then be determined by summing the individual responses from all individual asperities, but taking into account interaction between asperities.

To formulate a model in this way it is necessary to be able to model individual regular triangular asperities and the distribution of stresses throughout the profile. The theory that has been developed to simulate these processes are described briefly hereafter.

Single Asperity Model

The tests on flat, saw tooth and regular triangular profiles were used to develop basic theoretical models for asperity sliding and shearing.

Asperity Sliding

The direct shear tests confirmed the sliding models of Patton (1966) and Ladanyi and Archambault (1970) for unbonded purely frictional surfaces, i.e.

$$\tau = \sigma \tan(\phi_s + i)$$  (1)

where \(\tau\) is the sliding shear stress, \(\sigma\) is the average normal stress applied to the joint, \(\phi_s\) is the base friction angle of the joint and \(i\) is the asperity inclination. It should be noted that Eq. (1) was derived for rigid asperities, not the deformable asperities contained in the CNS test samples. The authors (Seidel and Haberfield, 1995b) adopted the energy approach used by Ladanyi and Archambault (1970) to show that Eq. (1) also holds for elastic materials. This result in itself is not startling, but as demonstrated later, it is important when considering the shear behaviour of irregular profiles consisting of multiple asperities of varying angle.
Asperity Shearing

After the initiation of interface slip, the contact area between the concrete and the rock gradually reduces from full contact area to smaller contact areas as shear displacement progresses. This is demonstrated in Fig. 3 for an interface comprising regular triangular asperities. Local normal stresses increase both as a consequence of the reduced contact area and as a result of the interface dilation in combination with the constant normal stiffness (CNS) condition. A critical normal stress is reached at which the asperity can no longer sustain the loading and individual asperity failure results. Observation of the video records clearly showed this failure to be rotational. This is in direct contrast to other models (e.g. Patton, 1966; Johnston and Lam, 1989) which are based on a flat failure surface.

The form of the failure planes observed in the direct shear testing, suggested the use of slope-stability methods to model the shear failure of the rock asperities. Sokolovsky’s (1960) closed form solution for the failure of weightless slope in a cohesive and frictional material subjected to an inclined load was therefore adopted. The assumption of weightlessness has little influence on the strength of the asperity and has the distinct benefit that the solution becomes scale independent. The Sokolovsky solution was found to give close estimates of both the shear stress and displacement at failure. In general, predicted values are within 5 to 10% of the experimental values (Seidel and Haberfield, 1998).

On the basis of the close predictions obtained, the slope stability analogy appears to be valid for predicting the peak failure strength of regular triangular asperities in weak rocks such as Johnstone. It is recognized that this failure mechanism will be affected in the natural rock by unfavourably oriented planes of weakness, and that the mechanism may not be applicable to rocks that degrade, such as limestone, or to harder rocks, where brittle failure rather than shear failure may occur.

Post-peak Behaviour

Another benefit of the Sokolovsky solution is that it also defines the geometry of the failure surface. Observation of the video camera records of the shear tests on profiles comprising only regular triangular asperities confirms the development of a curved failure surface emanating from the apex of the loading concrete asperity, and intersecting the trailing face of the rock asperity, as shown in Fig. 4. The
geometry of the observed failure surfaces, and in particular the location of the intersection point on the trailing rock face, were found to be predicted reasonably well by the Sokolovsky solution.

However, during the laboratory tests it was observed that further shear displacement did not occur on the curved failure surface, but rather on a chord linking the intersection points of the initial failure surface with the leading and trailing asperity faces (Fig. 4). The inclination of this chord, which defines the lower planar surface for post-peak shear movements, can be determined from the Sokolovsky solution.

Post-peak Friction Angle
The planar shear surface along which the rock wedge moves is entirely within failed material at residual strength. The failed rock between this shear surface and the concrete asperity is effectively a granular material, held in place by the high local confining stresses. Fig. 4 shows the sheared asperity after some post-peak displacement has already occurred, and it is evident that part of the zone of sheared material extends beyond the trailing face of the rock asperity, where it is unconfined, and spalls as debris down the trailing face.

Careful examination of the video records reveals that the zone of granular material exhibits a rolling motion at the contacts with both the concrete asperity and the rock shear plane. This is accompanied by relative motion both of the zone of granular material with respect to the intact rock, and of the concrete asperity with respect to the granular zone. Using an energy approach similar to Ladanyi and Archambault (1970), Seidel (1993) was able to derive a theoretical expression for the post peak sliding shear stress. The post peak friction angles determined from this expression are in close agreement with observed values. Of particular interest is that both the predicted and observed post-peak friction angles are in excess of the residual friction angle of the rock, despite the contraction of the interface in the post-peak phase.

Multiple Asperity Model
As argued earlier, the fractal model of roughness (Seidel and Haberfield, 1995a) allows irregular profiles to be simulated by a series of triangular asperities with constant side length but different asperity angles. The behaviour of this simplified profile can be determined by accounting for interaction effects between asperities and summing the responses from individual asperities. This approach has been used to extend the single asperity model to irregular profiles containing multiple asperities.

The CNS tests on irregular profiles (irregular triangular and fractal profiles), clearly demonstrated that asperities with different asperity angles could be in contact simultaneously. This occurs because the asperities are not rigid but deform under load. As a consequence, sliding will occur on not only the currently steepest asperity slope, but may also occur simultaneously on asperities of lower inclinations (Haberfield and Johnston, 1994). For the assumption of elastic asperities, the authors (Seidel and Haberfield, 1995b) were able to extend the energy approach mentioned earlier to show that for an elastic joint profile of \( n \) asperities of variable angles, \( i_j \), the global sliding shear stress will be given by

\[
\tau = \frac{1}{A} \sum_{j=1}^{n} \alpha_j \sigma_n \tan(\phi_v + i_j)
\]  

(2)
where \( A \) is the total joint contact area, and \( a_i \) and \( \sigma_{ij} \) are the contact areas of, and normal stresses acting on the individual asperities. Eq. (2) implies that the total sliding resistance of an irregular profile is the sum of the components of sliding resistances of individual asperities and is independent of the steepest asperity angle currently in contact, which governs the dilation of the interface.

Another aspect of multiple asperities of varying angle in contact is that the distribution of stresses between adjoining asperities becomes significantly more complex than the uniform distribution that exists with regular triangular asperity interfaces. On application of shear displacement to an irregular profile, there is a tendency for the sample dilation to be controlled by the steepest asperity angle, and for separation to occur between the interface halves on the less steep asperities. However, since the asperities must deform under load, compatibility of displacements forces the steeper asperities to deform more than the less steeply inclined asperities. This deformation pattern results in the steepest asperities attracting the highest stresses and the low inclination asperities attracting relatively little stress (Haberfield and Johnston, 1994). It follows therefore that this deformation pattern needs to be established to enable realistic modelling of irregular profiles. Once a method of determining asperity deformations has been established, it is a relatively simple matter, using matrix methods and enforcing global compatibility, to calculate the stresses applied to individual asperities (Seidel and Haberfield, 1998)

*Asperity Deformations*

The total deformation of individual asperities is derived from two components; the deformation due to stresses applied directly on an asperity's surface and the concomitant deformations resulting from loading of other surrounding asperities. The zone of influence for individual asperities depends on the geometry of the interface. For the CNS tests, the zone of influence will be one-dimensional, and rather large. For real joint planes, or drilled shafts, the representation of the load as a strip is dubious. Rather, it is more likely that the random roughness results in loaded "patches". In this case, the zones of influence of these patches will be two-dimensional, and therefore more limited, but intense. The deformed shape of the profile is determined using the approximate method of Steinbrenner (1934) (see Terzaghi, 1966). Steinbrenner's method is a widely known technique used by geotechnical engineers for the determination of the settlement of surface footings situated on an elastic layer of finite depth. The method is particularly attractive as a wide range of load width to depth ratios can be accommodated.

*Stress Distribution Determination*

As stated earlier, using established matrix methods, it is a reasonably simple matter to determine the distribution of stresses over the profile if asperity deformations are known. By applying a unit stress to each asperity in turn, a vector of compliance values can be established to relate displacements at every other asperity to the applied unit stress. A global compliance matrix can then be compiled for the entire profile. The stresses however, are not known but must be determined. This requires a reformulation of the problem by inversion of the compliance matrix to give a stiffness matrix relating load on any asperity to a unit displacement of any other asperity. The relative displacements of the asperities are fixed by geometrical considerations, and the absolute displacements can be determined using the equilibrium equation established by the CNS condition which relates global normal stress to dilation.
Comparison with Laboratory Test Results

The analytical models described above have been incorporated into a computer program. Comparisons between model predictions and the CNS direct shear tests are detailed below. These predictions are definitely not "Class A" predictions (Lambe, 1973); they are made in hindsight, with the tests on regular triangular asperities forming the basis for the models which have been adopted. However, since the various components of the models, which cover asperity sliding, shearing and elastic distribution of stresses are based only on theoretical concepts, it does not matter that the predictions are made *a posteriori*.

The parameters required for input into the model are restricted to basic rock strength properties (i.e. cohesion, peak and residual friction angles, Young's modulus and Poisson's ratio), fractal dimension of the roughness profile, chord length, and the boundary conditions (initial normal stress and normal stiffness). All predictions have been made using the same average rock properties.

Typical comparisons between actual and predicted responses of two CNS tests on regular triangular asperity profiles are presented in Fig 5. The shear stress, normal stress and dilation vs shear displacement predictions are compared in a single graph for each test. The left-hand Y-axis is in stress units; shear stress and normal stress plots can be referred to this axis. In addition, the right-hand Y-axis is in length units, and the interface dilation can be determined from this axis. Because of the relationship between normal stress and dilation in a CNS test, both normal stress and dilation can be evaluated from a single curve (using left and right axes respectively).

![Graphs showing shear and normal stress vs shear displacement](image)

**Figure 5** Predicted vs measured performance of regular triangular asperity profiles

Although predictions have been carried out for all tests (which include regular triangular asperities varying from 5° to 27.5° and a range of stress and stiffness conditions), only comparisons for 5° (600 kPa/mm normal stiffness and 600 kPa initial normal stress – shown on the left) and 15° (300 kPa/mm normal stiffness and 300 kPa normal stress – shown on the right) regular triangular asperity profiles are included here. Similarly close comparisons were obtained for all of the other tests on regular asperity profiles. Fig. 6 compares the predicted and measured peak shear stresses for all tests on regular triangular asperities and shows that in general that agreement was obtained within ±10%, which is within observed experimental
scatter. Figs. 5 and 6 clearly demonstrate that the sliding and shearing models adopted capture the essential behaviour of regular triangular asperity profiles.

Fig. 7 follows the convention of Fig. 5, plotting observed and predicted behaviour of two fractal profiles (designated Class B and Class C) at a stiffness of 300 kPa/mm and an initial normal stress of 300 kPa. Predictions agree well with the measured response. Fig. 8 compares the measured and predicted peak shear stress for all tests on the fractal interfaces. All predictions lie within ±10% of the observed values. Figs. 7 and 8 clearly show that the behaviour of complex interface geometries can also be modelled by idealising these profiles as a series of triangular asperities of variable angle but constant side length and accounting for elastic interactions between asperities in a rational way.

**Figure 7** Predicted vs measured performance of two fractal profiles

**Figure 8** Measured vs predicted peak shear stress for fractal profiles

**Rock/rock joints**

The success of the concrete/rock research prompted the authors to extend the work to include natural rock/rock joints. A total of 45 direct shear tests have been conducted to date on Johnstone and sandstone joints containing both regular triangular and fractal profiles. Other parameters investigated include normal stress and stiffness, roughness and scale. From observations of the direct shear tests it became apparent that several significant
differences exist between concrete/rock and rock/rock interfaces. Of particular note was the significantly reduced strength of rock/rock joints when compared to that of identical concrete/rock joints (see Fig. 9).

(a) Regular triangular asperity profile  
(b) Fractal profile

Figure 9  Comparison of rock/rock and concrete/rock behaviour

Tests on regular triangular asperity profiles revealed that the failure mechanism for Johnstone/Johnstone joints was significantly different from concrete/Johnstone joints. For concrete/Johnstone joints, the much stronger concrete half of the joint constrained failure over the full contact length of each asperity. However for Johnstone/Johnstone joints the material on both sides of the interface is similar, allowing failure to occur at localised regions of high stress. Failure gradually progressed until complete failure of each asperity (and therefore of the whole interface) occurred. This resulted in a significant reduction in strength. The finite difference program, FLAC, was used to determine the stress distribution over the rock/rock interface. Based on this stress distribution, new theoretical algorithms for asperity failure are being developed. Other minor differences in behaviour have also been identified and appropriate theoretical models are being developed.

Tests on sandstone/sandstone joints indicated a fundamental difference in interface shear behaviour from the Johnstone/Johnstone joints. Failure of Johnstone asperities for both concrete/rock and rock/rock joint interfaces occurred on a curved surface, resulting in the development of the sliding/shearing model. However for sandstone a wearing mechanisms appears to be the dominant form of failure. On the basis of shear test results a degradation rate approach is being developed to model the shear response of sandstone/sandstone joints. Recent tests on limestone joints show similar behaviour to the sandstone joints.

Conclusions

This paper briefly describes the analytical models developed to predict the shear behaviour of concrete/rock interfaces. The analytical models include models for sliding between rough surfaces of essentially elastic materials, asperity shearing and post-peak behaviour and elastic stress distributions. Underlying these new models is a strongly held philosophy of the importance of understanding the fundamental mechanisms of rough interface behaviour, rather than resorting to empiricism. These models have been combined into a computer program for the analysis of the shear
behaviour of concrete/rock interfaces. The predictions have been shown to compare well with the results of direct shear tests on interfaces containing regular triangular and fractal roughness profiles.

Results of direct shear tests on rock/rock joints show significantly lower peak strength than measured with concrete/rock joints of the same roughness geometry and rock strength. Reasons for these differences have been identified, and appropriate models of behaviour are currently being developed.

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References


