Comparison of Shear Behaviour under Direct Shear and Constant Normal Stiffness Conditions

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Abstract: The shear behaviour of modelled saw-tooth and tension joints was investigated in the laboratory under Constant Normal Stiffness (CNS) condition using a large-scale shear apparatus, developed in-house. Test results obtained under CNS condition are compared with the conventional direct shear tests, that follow a Constant Normal Load (CNL) condition. It is observed that the peak shear stress obtained under CNL condition is always smaller than the CNS condition, especially for the natural (tension) joints. The normal stress increases with the shear displacement under CNS condition, whereas it remains unchanged for the CNL condition. The dilation of the joints under CNL condition is much greater in comparison to CNS testing. The strength envelope for CNL shows an upper bound for all the tests, and the envelope for CNS provide a lower bound. Tests on bentonite filled saw-tooth joints show a critical infill thickness to asperity height ratio ($t/a$) of 1.4. A mathematical model is proposed to predict the shear behaviour of soft joints especially filled with gauge material. A computer program is also outlined to estimate the peak shear strength of clean and infilled joints. In comparison to the laboratory results, the proposed model predicts the shear strength very closely.

Key Words: infilled joint, modelling, normal stiffness, shear strength, soft joint

Introduction

The shear behaviour of rock joints is generally investigated in the laboratory using the direct shear apparatus where the normal load remains unchanged during shearing. This is acceptable for planar joints where the dilation associated with shearing is negligible. However, shearing of non-planar joints such as saw-tooth and natural joints results in considerable dilation, and if the surrounding rock mass does not deform sufficiently, then an inevitable increase in normal stress occurs. Therefore, in reality, shearing of non-planar joints takes place under Constant Normal Stiffness (CNS) condition rather than under CNL condition. In order to investigate the shear behaviour of joints, a large-scale shear apparatus was designed at University of Wollongong by the authors which could be used to test specimens under both CNL and CNS conditions. In the past, CNS tests have been conducted on much harder joints produced from gypsum plaster and/or concrete by several researchers [e.g. Benmokrane & Ballivy, 1989; Skinas et al., 1990; Ohnishi & Dharmaratne, 1990]. This work has been extended to the field of rock socketed piles by Lam & Johnstone (1989). Nevertheless, very few
research studies have been conducted on soft joints sheared under CNS condition. In order to differentiate the two types of testing methods, a detailed laboratory testing program has been undertaken by the authors, to investigate in detail the effect of CNL and CNS conditions on the shear behaviour of modelled soft joints.

The shear behaviour of the joints changes considerably when it is filled by gauge material such as clay and silt. Various infill materials influence the shear behaviour differently, for instance, a small thickness of clay fillings decreases the joint strength to that of the filling itself. If the infill material is silt, a much larger infill thickness is required to bring the shear strength down to that of the pure silt. In the past, laboratory tests have been conducted on infilled joints by several researchers, but mainly under CNL [de Toledo & de Freitas, 1993; Papaliangas et al., (1993); Phic-Wej et al., 1990; Pereira, 1990; Bertacchi et al., 1986; Barla et al., 1985; Lama, 1978; Ladanyi & Archambault, 1977 and Goodman, 1970]. It can be concluded that the infill thickness to asperity height ratio is closer to unity for cohesive infill, whereas for granular soil, this ratio approaches two. However, there are a very limited number of studies [e.g. Cheng et al., 1996 and Indraratna et al., 1998] which directly investigate the influence of infill on the shear behaviour of soft rock joints under CNS. Cheng et al. (1996) concluded from tests conducted on concrete/rock joints that the shear response is purely frictional and is independent of infill thickness. Indraratna et al. (1998) pointed out that the joint strength becomes the same as that of the infill material (bentonite), when the infill thickness to asperity height reaches 1.40. Considering the very limited research on infilled joints under CNS condition, the current laboratory testing program has been extended, whereby regular saw-tooth shaped infilled joints have been used for simplicity of modelling, both experimentally and mathematically.

**Applicability of the CNL and CNS Conditions**

Figure 1 shows an underground excavation where potentially unstable rock blocks are constrained between two parallel dilatant rock joints. The sliding of such blocks inevitably increases the normal stress, and also, dilation becomes significant if the joint surfaces are rough. The increase in normal stress on the shear plane is equal to \( k \cdot dv \), where \( k \) is the stiffness of the surrounding rock mass and \( dv \) is the dilation. For this case, tests conducted under Constant Normal Load (CNL) condition will yield shear strengths that are too low (Goodman, 1976). Another example, defined in Figure 2a shows a rock socketed pile where the interface between the concrete and the socket is considered to be rough. When this pile is loaded vertically, the side shear resistance develops as a function of the variable normal stress associated with the dilation of the rough joint surface. The deformation mechanism and the simplified 2-D models are given in Figures 2 (b, c, d).

In general, CNL condition is only realistic for shearing of planar interfaces where the normal stress applied to the shear plane remains relatively constant such as in the case of rock slope stability problems. However, for situations as illustrated in Figures 1 and 2, the development of shear resistance is a function of the constant normal stiffness (CNS), and the use of CNL test results for such situation leads to underestimated shear strengths.
Preparation of Soft Joint

Soft joints are prepared from gypsum plaster (CaSO4·H2O hemihydrate, 98%). The plaster is mixed with water in the ratio of 5:3 by weight and poured inside the shear box to obtain fully mated joints, having asperity inclinations of 9.5°. Two distinctly different surface profiles are used: (i) saw-tooth profile and (ii) natural, tension joint profile obtained from a sandstone specimen fractured under Brazilian test. The surface profiles of the two specimens were drawn using a digital co-ordinate measuring apparatus (Figure 3). The bottom specimen was cast inside the bottom mould containing the required surface profile and left for an hour to cure. The top specimen was then cast on the bottom specimen, and subsequently, the whole assembly was left for another hour under room temperature. The moulds were then stripped and kept inside an oven for 14 days at a controlled temperature of 50° C. The mechanical properties of the cured plaster were determined by conducting tests on 50 mm diameter specimens. The test results showed an uniaxial compressive strength ($\sigma_c$) of 11 to 13 MPa and a Young’s modulus (E) of 1.9 to 2.3 GPa. This model material is found to be suitable for modelling a number of sedimentary soft rocks such as coal, friable limestone, clay shale and mudstone based on dimensionless strength factors (Indraratna, 1990).

Commercial bentonite was used as an infill material for the simulated soft joints. A placement moisture content of 12% was maintained for all the tests. Direct shear tests were performed on infill material for a wider range of normal stresses, and a peak friction angle of 35.5° was obtained. It is reported by Phien-wcj et al. (1990) that bentonite is appropriate to simulate the behaviour of frictional infill material. The preparation of the infill joint was completed in three steps: (i) fixing the adjustable
collar, (ii) filling with infill and (iii) placing the mould inside the outer box of the machine. A close view of the prepared infilled joint is shown in Figure 4.

![Diagram](image)

(a)  

(b)  

Figure 3. Surface profile of (a) saw-tooth joint, \( i=9.5^\circ \) and (b) tension joint.

![Photograph](image)

Figure 4. Photograph of a prepared saw-tooth infilled joints.

All tests were conducted in a large-scale shear apparatus which consists of two top and bottom boxes of size 250x75x150mm and 250x75x100mm, respectively. The top box can only move vertically on ball bearings, and the bottom box which is rested on bearings can move only in the direction of shearing. The normal stiffness of the rock mass is modelled in the laboratory by an assembly of four springs placed on the top of specimen. The normal and shear loads are applied through hydraulic jacks. The piston of the shear loading jack is connected to a strain controlled device which operates under a prefixed strain rate, within the range of 0.30 to 1.70 mm/min. The apparatus has normal and shear load capacities of 180 kN and 120 kN, respectively. Load cells are connected with digital strain meters to monitor the change in normal and shear loads during shearing. In order to measure the dilation and horizontal displacements, strain gauges were located on the center of the top specimen, and also along the shear direction.
Laboratory Tests

CNL and CNS tests were conducted on saw-tooth joints (inclination of 9.5°), and on natural tension joints, using initial normal stress ($\sigma_{\text{in}}$) ranging from 0.16 to 2.43 MPa. All specimens were sheared at a strain rate of 0.50 mm/min under a Constant Normal Stiffness (k) of 8.5 kN/mm. Saw-tooth infilled joints were sheared under initial normal stresses of 0.30 and 0.56 MPa, for the infill thickness to asperity height ratio ($t/a$) varying from 0 to 1.80.

Results and Discussions

Effect on the Shear Behaviour of Joints

The shear behaviour of the tension and saw-tooth joints under both CNS and CNL conditions is plotted in Figure 5. The left hand side of Figure 5 shows the shear behaviour of tension joints and the right hand side shows the behaviour of saw-tooth joints. It is observed that CNL condition always underestimates the peak shear stress of the joints. Also, at higher stresses, CNS tests indicate a more strain softening behaviour. For both types of joints, for a given displacement, the shear stresses are greater under CNS condition, which is attributed to the increased normal stress during shearing. Similar results have been reported by Skinas et al. (1990) and Ohinishi & Dharmaratne (1990) for much harder joints.

Effect on Normal Stress

The normal stress increases as one asperity overrides another. At very low $\sigma_{\text{in}}$, the increase in normal stress is more pronounced, but at high $\sigma_{\text{in}}$ values, little variation of normal stress is observed during shearing. This suggests that at high $\sigma_{\text{in}}$, the CNS behaviour approaches CNL. In other words, the asperities are sheared at very high normal stresses, thus indicating flatter normal stress vs horizontal displacement curves (Fig.6). For non-planar joints tested in this study, the normal stress is observed to increase almost by the amount of $k.d_{w}$ with the shear displacement, for both the saw-tooth and tension joints.

Effect on Dilation

The variations in dilation with horizontal displacements are measured for both CNL and CNS conditions as plotted in Fig. 7. It is observed that the CNL condition overestimates the dilation of the joints in comparison with the CNS condition. The dilation resulting during CNS is smaller, because the normal stiffness representing the overlying rock mass resist joint dilation to some extent. As expected, at low $\sigma_{\text{in}}$ values, the dilation vs displacement curves take the initial shape of the asperity (ic. fully elastic behaviour).
**Effect on Strength Envelopes**

In order to obtain the peak shear stress envelopes for the two type of joints, the corresponding variations in peak shear stress versus normal stress for different $\sigma_{ob}$ are plotted in Figure 8. It is evident that CNL envelope for saw-tooth joint is bilinear, and it represents an upper bound for all the tests. In contrast, the CNS envelope can be described as linear for this particular joint type. However, the peak strength envelope may be nonlinear for higher asperity angles. For the natural (tension) joints, the CNL and CNS strength envelopes are both linear for the range of normal stresses employed. The CNL envelope is also observed to serve as an upper bound for the tension joints, thereby providing larger shear strength parameters. Therefore, in reality, CNS test data
may produce a more reliable angle of shearing resistance as applicable to the design of excavations in jointed rocks.

Figure 7. Dilation behaviour of saw-tooth (S1) and tension joint under CNS and CNL.

Figure 8. Strength envelopes for saw-tooth and tension joints under CNL and CNS.
Tests on Infill Joints

Shear tests were conducted on infilled saw-tooth joints for infill thicknesses ranging from 0 to 4.5 mm, which correspond to a t/a ratio of 0 to 1.80. The shear stress, normal stress and dilation responses to horizontal displacements were recorded at every 0.5 mm interval, until the peak to peak contact was attained. Test data indicate that the peak shear stress decreases significantly in contrast to clean joints as a result of the addition of a thin infill layer, 1.5 mm in thickness. As the infill thickness approaches the asperity height (i.e. t/a=1.0), the shear stress and normal stress responses remain generally unchanged (Fig. 9). This indicates that the effect of asperity is diminished and that the shear behaviour is mainly governed by the infill. The shear stress attains the peak rapidly as the infill thickness is increased further. Figure 9 shows that the shear stress and normal stress curves represent the behaviour of the infill alone, as the infill thickness exceeds 2.5 mm. The joints undergo a compressive behaviour for an infill thickness greater than 2.5 mm (Fig. 9c, f). Therefore, it may be concluded that the change from dilatant to compressive behaviour of joints occurs as the critical infill thickness to the asperity height ratio is exceeded. In this study, a t/a ratio of 1.40 may be considered as critical, beyond which the shear strength of joint can be modelled by the strength of the infill itself.

Mathematical Model for the Prediction of Shear Strength of Soft Joint

Characterisation of Joint Dilation

Fourier transform technique (Spiegel, 1974) has been used in this study to characterise the joint surfaces before and after shearing of the specimen. The successful application of the Fourier series in the field of rock mechanics is not yet established. Fourier series is usually used to define a continuous function \( f(x) \) which is integrable along the period \( 2\pi \), and has an integrable derivative at some interval \((a, b)\). The following form of Fourier series is adopted here to characterise the joint profile for a prescribed period, \( T = b - a \):

\[
\delta_s(h) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{2\pi nh}{T}\right) + b_n \sin\left(\frac{2\pi nh}{T}\right) \right]
\]

(1)

where,

\[
a_n = \frac{2}{T} \int_{a}^{b} f(x) \cos\left(\frac{2\pi nx}{T}\right) \, dx, \quad \text{and} \]

(1a)

\[b_n = \frac{2}{T} \int_{a}^{b} f(x) \sin\left(\frac{2\pi nx}{T}\right) \, dx
\]

(1b)

The Fourier series is calibrated to match the exact joint dilation with horizontal displacement, where the coefficients \( a_n \) and \( b_n \) are determined from the experimental data. For the current test data, the Fourier coefficients are calculated for Type I joints before and after shearing under different initial normal stresses (\(\sigma_{iso}\)) and T=30 (Table 1).
Figure 9. Behaviour of saw-tooth (S1) infilled joints under CNS condition.
Table 1. Fourier coefficients for saw-tooth joints for various $\sigma_{No}$

<table>
<thead>
<tr>
<th>$\sigma_{No}$ MPa</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
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<tr>
<td>initial profile</td>
<td>2.5</td>
<td>-1.036</td>
<td>0</td>
<td>-0.137</td>
</tr>
<tr>
<td>0.16</td>
<td>1.655</td>
<td>-0.657</td>
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<td>-0.035</td>
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<td>1.437</td>
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<td>-0.053</td>
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<td>-0.057</td>
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<tr>
<td>1.63</td>
<td>0.968</td>
<td>-0.347</td>
<td>-0.059</td>
<td>-0.027</td>
</tr>
</tbody>
</table>

Variation in Normal Stress

Once the joint dilation $[\delta_v(h)]$ with horizontal displacement $h$ under a given initial normal stress $\sigma_{No}$ is fitted to the Fourier series (Eqn. 1), the variation of normal stress under constant normal stiffness $k_n$ can be determined by Eqn. 2:

$$\sigma_n(h) = \sigma_{No} + \frac{k_n \cdot \delta_v(h)}{A} \tag{2}$$

where, $\sigma_n(h)$ = normal stress at any horizontal displacement, $h$; $\sigma_{No}$ = initial normal stress; $k_n$ = normal stiffness; $\delta_v(h)$ = dilation corresponding to horizontal displacement, $h$; $A$ = joint surface area.

Prediction of Shear Stress

Unlike in many previous joint models where the asperity angle (i) was regarded as constant, the Fourier analysis permits degradation of the asperity angle as a function of horizontal displacement, $i(h)$. Subsequently, the shear stress response with the horizontal displacement can be calculated from Eqn. 3 as given below:

$$\tau(h) = \sigma_n(h) \tan(\phi_b + i(h)) \tag{3}$$

where, $\sigma_n(h)$ is given by Eqn. 2; $\phi_b$ = basic friction angle; $i(h)$ = inclination of the tangent to the dilatancy curve at any horizontal displacement, $h$.

Considering the fact that both $\sigma_n(h)$ and $i(h)$ are continuous functions, the solution for 'peak' shear stress ($\tau_p$) always exists, and it can be numerically determined using a computer subroutine.

Peak Strength Drop ($\Delta\tau_p$) due to Infill

In order to predict the drop in peak shear stress due to infill material, an empirical relationship can be established from the laboratory tests data, following the hyperbolic model proposed earlier by Duncan & Chang (1970):
\[
\left( \frac{\Delta \tau_{\text{peak}}}{\sigma_{\text{no}}} \right) = \frac{t/a}{\alpha(1/a) + \beta} \]

where, \( \alpha \) and \( \beta \) = constants depending on \( \sigma_{\text{no}} \) and surface roughness. The values of \( \alpha \) and \( \beta \) are 0.51 and 0.13 for \( \sigma_{\text{no}} = 0.30 \) MPa and 0.71 and 0.18 for \( \sigma_{\text{no}} = 0.56 \) MPa.

**Infill Joint Shear Strength**

The peak shear strength of infilled joint under CNS condition can be calculated according to Eqn. 5, once the strength of clean joints is known at a given \( \sigma_{\text{no}} \) for a particular joint profile.

\[
\left( \tau_p \right)_{\text{infill}} = \left( \tau_p \right)_{\text{clean}} - \Delta \tau_p
\]

where, \( \Delta \tau_p \) = drop in peak shear stress due to the inclusion of infill.

The flow chart shown in Figure 10 illustrates the step by step procedure for calculating the shear strength of infilled joints under CNS conditions by using a computer code.

**Verification of the Proposed model**

The proposed mathematical model is used to predict the shear behaviour of the saw-tooth clean and infilled (bentonite) joints under CNS condition. The Fourier Coefficients, surface profile properties, initial normal stress, normal stiffness, basic friction angle and \( t/a \) ratio are used as input parameters in the computer program. The predicted outputs include the shear stress with displacement, normal stress variation, peak shear stress and infill joint strength for various \( t/a \) ratios. A brief comparison of the experimental and predicted results is summarised in Table 2. It is observed that the model prediction for peak shear stress and normal stress for the clean joints is in good agreement with the experimental results. The predicted peak shear strength for infilled joints under the applied initial normal stresses and various \( t/a \) ratios is also observed to match the experimental value, within acceptable accuracy.

<table>
<thead>
<tr>
<th>( \sigma_{\text{no}} )</th>
<th>( \tau_{\text{peak}} )</th>
<th>( \sigma_n )</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Expt</td>
<td>Model</td>
</tr>
<tr>
<td>0.16</td>
<td>0.48</td>
<td>0.45</td>
</tr>
<tr>
<td>0.30</td>
<td>0.65</td>
<td>0.57</td>
</tr>
<tr>
<td>0.56</td>
<td>1.00</td>
<td>0.91</td>
</tr>
<tr>
<td>1.10</td>
<td>1.54</td>
<td>1.33</td>
</tr>
<tr>
<td>1.63</td>
<td>1.80</td>
<td>1.65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t/a )</th>
<th>( \tau_p (\sigma_{\text{no}}=0.30 \text{ MPa}) )</th>
<th>( \tau_p (\sigma_{\text{no}}=0.56) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ratio</td>
<td>Expt</td>
<td>Model</td>
</tr>
<tr>
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<td>0.65</td>
<td>0.57</td>
</tr>
<tr>
<td>0.6</td>
<td>0.31</td>
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</tr>
<tr>
<td>1.2</td>
<td>0.17</td>
<td>0.11</td>
</tr>
<tr>
<td>1.4</td>
<td>0.16</td>
<td>0.10</td>
</tr>
<tr>
<td>1.8</td>
<td>0.15</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Table 1. Fourier coefficients for initial asperity angle ($\theta_i$) and normal stress ($\sigma_{no}$)

INPUT

Fourier Coefficients for
initial asperity angle ($\theta_i$) and
normal stress ($\sigma_{no}$)

Estimate dilation by Fourier series:

$$\delta_i(t) = \frac{\theta_i}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos \left( \frac{2\pi nt}{T} \right) + b_n \sin \left( \frac{2\pi nt}{T} \right) \right]$$

Calculate slope of tangent ($i_n$) at any shear displacement, $h$:

$$i_n = \tan^{-1} \left( \frac{d\delta_i}{dh} \right)$$

INPUT

normal stiffness, $k_n$

Estimate the normal stress at any shear displacement, $\sigma_n(h)$:

$$\sigma_n(h) = \sigma_{no} - \frac{k_n \cdot \delta_i(h)}{A}$$

INPUT

basic friction angle, $\phi_b$

Estimate shear stress ($\tau_n$) at any shear displacement, $h$:

$$\tau_n = \sigma_n(h) \times \tan(\phi_b + i_n)$$

Find $\tau_{peak}$

Is the joint is infilled? (YES/NO)

YES

Calculate drop in peak shear stress, $\Delta\tau_{peak}$

$$\Delta\tau_{peak} = \sigma_{no} \left[ \frac{1}{\sigma_{no} \times \cos(\theta_i) + \beta} \right]$$

NO

Estimate infilled joint peak shear stress, $\tau_{peak\text{, infill}}$

$$\tau_{peak\text{, infill}} = \tau_{peak\text{, dem}} - \Delta\tau_{peak}$$

Figure 10. Flow chart showing the outline of the computer program.
Conclusions

The shear behaviour observed under Constant Normal Stiffness (CNS) condition is distinctly different from the conventional direct shear test or Constant Normal Load (CNL) condition for non-planar joints. The shear strength observed under CNS is higher than that of CNL. In the case of CNL, the dilation of joints is overestimated, hence, the shear strength is underestimated. The strength envelope under CNL condition may be considered as the upper bound of all the tests. The effect of infill on the shear behaviour of saw-tooth joints reveals that a t/a ratio of 1.40 may be considered as critical for bentonite filled joints, beyond which the joint behaviour becomes compressive rather than dilatant and the shear strength of the joint becomes the same as that of the infill material. A mathematical model is proposed to predict the shear strength of clean and infilled soft joints. It is verified that the model predictions are in acceptable agreement with the laboratory results, in relation to the peak shear stress (shear strength) of both clean and infilled joints.

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References


