Bolt Pretension of an Idealised Underground Bedded Roof Formation

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Abstract: Bolting of a number of strata of an underground roof, with tensioned and fully grouted cable bolts, increases the capacity of the roof structure if compared to the capacity of an individual single stratum. The stability of such bolted vertically jointed or cracked horizontal roof strata with various spans and strata thicknesses and various degrees of pretensioning and spacing of the cable bolts is investigated with a discontinuum numerical code that allows for the development of small or large deflections before any collapse of the bolted beams. Thus, the roof deflection, any opening of the joints and the distribution of the stress along an interlayer and across two critical sections of each of the tested rock beams is investigated and illustrated. These stratified roof rock beam responses are compared to those of roofs composed of intact rock.

Keywords: voussoir beam, cable bolt, stratified rock, discontinuum modelling

Introduction

Roofs of underground openings in sedimentary environments are usually conformable with the bedding planes. The latter have usually low tensile strength in their normal direction and a limited shear strength of their surfaces. Natural cross joints or induced transverse fractures do not allow for the simulation of roof rock strata as continuous beams. Voussoir beam theory assumes the existence of cracks within the single layered beam, thus disallowing the development of tensile stresses within it. The stability against bending of such a beam depends on a couple of equal thrust forces at two cross sections and a changing with distance from the abutment lever arm of the thrust line. Evans (1941) pioneered in the analysis of such beams, whereas Sterling (1980) investigated such beams on physical models in the laboratory. Brady and Brown (1985) presented then a state of the art on the subject. Sofianos et al. (1995) and Sofianos (1996) developed and validated numerically a closed form solution for the response of single layer voussoir beam roofs.

Bolting of a number of strata with prestressed cable bolts increases the capacity of the roof structure, by transforming it into an equivalent beam with larger thickness and initial lever arm, and less deflection than the individual single stratum. Wright (1974) investigated such bolted roof structures with continuum finite element models. Beer and Meek (1982) suggested that the equivalent thickness of a bolted multilayered beam may be considered to be half to two thirds of the total thickness of the bolted layers, depending on the amount of pretension and the spacing of the bolts.

In order to investigate further the effect of the pretension on the stability of such bolted jointed roof rock strata, numerical simulation of the behaviour of such beams is performed. For this reason, a number of span and strata thickness rock beam configurations with various degrees of bolt pretension, are tested with a numerical code that allows for the development of large deflections and for the opening of the discontinuities.

Modelling

The Distinct Element Method, which is a discontinuum numerical procedure developed by Cundall (1971), is considered to be the most suitable to simulate the behaviour of such jointed rock. Its 2-D numerical implementation is performed with the computer code UDEC®(1993). This allows for the development of large displacements before any collapse of the beam. Thus, the process of deflection of the beam and opening of the joints may be simulated and the distribution of the stresses within the rock may be established. The symmetry of the beam allows for the simulation of its half only. Each model consists of three main blocks, one deformable in the centre, which simulates the half beam and two rigid, one at each side of the half beam. The left one simulates the abutment and the right one imposes the appropriate boundary conditions at the midspan.

The deformable block, shown for a particular case in Fig. 1, is subdivided into a mesh of 50X20 finite difference elastic elements which are allowed to slip or separate along interlayers, and contains bolts that tighten the strata together. The contacts between the deformable block and the rigid blocks are treated as discontinuities and are represented as a normal and shear stiffness between two planes which may

![Figure 1. Distorted view of the discretization of the stratified roof half beam, with s=20m, \( t_s = 0.50 \text{m}, h_b = 2.0 \text{m}, s_b = 1.0 \text{m.} \)
contact one another. Large displacements along discontinuities and rotations of blocks are allowed. In the normal direction the stress-displacement relation is assumed to be linear and governed by the normal stiffness $k_n$ so that:

$$\Delta \sigma_n = k_n \Delta u_n$$  \hspace{1cm} (1)

where:
- $\Delta \sigma_n$ = effective normal stress increment
- $\Delta u_n$ = normal displacement increment.

Similarly in shear the response is controlled by a constant shear stiffness $k_s$. The shear stress $\tau_s$ is limited by a combination of cohesion $C$ and frictional angle $\phi$ strength. Thus, either:

$$|\tau_{12}| \leq C + \sigma_n \cdot \tan \phi = \tau_{\text{max}} \Rightarrow \Delta \tau_s = k_s \cdot \Delta u_{s}^e$$

or

$$|\tau_{12}| \geq \tau_{\text{max}} \Rightarrow \tau_s = \text{sign}(\Delta u_s) \cdot \tau_{\text{max}}$$  \hspace{1cm} (2)

where:
- $\Delta u_s^e$ = elastic component of the incremental shear displacement
- $\Delta u_s$ = total incremental shear displacement.

In the right hand discontinuity, which represents the middle of the beam, both vertical slip and lateral separation are permitted. This is achieved by imposing zero friction angle combined with very large normal stiffness. In the left hand discontinuity, which represents the beam abutment, only separation is permitted. This is achieved by imposing very large values for normal stiffness, shear stiffness and friction angle. Between the individual strata there may be no tensile stress, whereas the shear stress is limited by the bedding plane friction angle.

Cable bolts are fully grouted. They may not transmit dowel stresses across their section, thus allowing for the transmission of axial stresses only along their length. Pretension of the bolts is applied first followed by grouting, prior to any deflection of the stratified beam. The bolts have an equivalent diameter 20mm, a tensile strength 120kN and an equivalent elastic modulus 100GPa.

**Bolted beam**

The roof rock beam, such as shown in sketch in Fig. 2, is assumed to be composed of individual beam layers that are bolted together. The modulus of deformation of the rock mass is taken 8GPa and the Poisson’s ratio $\nu$ as zero. The interlayers between the individual strata are considered to have a friction angle of 30° and zero cohesion. Three pairs of spans $s$ and pertinent layer thicknesses $t_i$ are considered, as given in columns 1 and 2 of table 1. The individual layers of each span are tied together with cable bolts of length $l_b$, 2m. A square pattern is adopted for the arrangement of the bolts, with a spacing $s_0$ of 1, 1.5 and 2 m. These bolts are pretensioned with a force $P_b$ of 0 (no pretension), 20, 50 and 80 kN, and then are fully grouted. In table 2 the various bolt parameters are given. These provide for each of the three span/thickness
pair configurations 12 beam cases to be tested. Loading of each of the voussoir rock beams is assumed to be due to only its own unit weight $\gamma$ of 30 kN/m$^3$.

Figure 2. Stratified roof rock beam tied with pretensioned bolts

| Table 1. Deflection and extreme strain for three span configurations |
|---|---|---|---|---|---|---|---|
| $s$ [m] | $t_b$ [m] | $\delta_{to}$ | $\delta_{ho}$ | $\delta_{nb}$ | $\sigma_{o}$ [MPa] | $\sigma_{b}$ [MPa] | $\sigma_{m}$ [MPa] |
| $P_b<0$ | $t=t_b$ | $t=t_b$ | $t=t_b$ | $t=t_b$ | abut. | mids. | abut. |
| 5 | 0.10 | 0.28 (0.27) | 0.160 | 0.001 (0.001) | 27.9 (34.1) | 23.1 | 2.3* | 2.2 | 0.40 (1.8) | 0.2 |
| 10 | 0.25 | 0.27 (0.27) | 0.135 | 0.004 (0.004) | 54.1 (74.4) | 45.5 | 6.9 | 4.7 | 1.66 (4.4) | 1.3 |
| 20 | 0.50 | unstbl. | 0.386 | 0.027 (0.027) | unstbl. | 49. | 30. | 7.70 (16.2) | 6.5 |

* non reliable result

| Table 2. Bolt parameters |
|---|---|---|---|
| no | variable | 1 | 2 | 3 | 4 |
| 1 | $P_b$ [kN] | 0 | 20 | 50 | 80 |
| 2 | $s_b$ [m] | 1 | 1.5 | 2 | - |
| 3 | $l_b$ [m] | 2 | - | - | - |

Deflection

In column 3 of Table 1, the analytically evaluated (according to formulae provided by Sofianos and Kapenis, 1998) normalised deflection $\delta_{en}=\delta_{n}/t_b$ is given for each of the three beam spans considered to have single layer thickness $t_b$; note that the third single layer configuration is unstable. In column 5, the analytically evaluated
normalised deflection $\delta_{sh} = \delta_h/t_h$ is given for each of the three beam spans considered to have thickness equal to the bolt length $l_b$. Respective numerical evaluations are given, below the analytical ones, in parentheses. The deflection of the bolted beams is anticipated to lie between these two extreme values; the largest deflection should be for the untensioned bolts. In column 4, the numerically evaluated normalised deflection $\delta_{sh} = \delta_h/t_h$ is given for the three spans considered for zero pretension force; the evaluated deflections are almost identical for the three bolt patterns considered for each span. It may be observed that these deflections are lower than those of the pertinent beams with single stratum thickness, whereas the numerical simulation evaluated a developing tension in the bolts with increasing deflection.

In Figure 3 the deflection of the various tested voussoir beams is related to the applied pretension on the beam by the cable bolts. The strata pretension is the ordinate and is defined as the applied bolt pretension $P_b$ divided by its tributary area $s_b^2$. The normalised roof deflection $\delta_h$ is the abscissa and is defined, as the deflection $\delta$ at midspan divided by the thickness of the individual strata $t_h$. Two linear, weakly decreasing, trendlines of the deflection with increasing pretension, may be observed. However, the case with $s=5m$, $t_e=0.10m$ shows a rapidly increasing deflection with decreasing pretension below 20kPa.

Shear stress along an interlayer

In Figures 4 the distribution of the shear stress along a lower half beam interlayer is illustrated for the three spans with the particular case of $P_b=50kN$ and $s_b=1m$. In these drawings the ordinate is the beam axis $x$, whereas the abscissa is the shear stress $\tau$ along the interlayer. In the same drawing the factor of safety against sliding is drawn, for which a secondary abscissa is drawn to the right. The value of the latter along the interlayers, unless the contact is lost, may be seen to be very close to one, which indicates slippage.
In figure 4a the span is 5m and the shear stress distribution is shown for the one but the lowest interlayer at 0.20m above the roof surface. Mountains at bolts (at $x=0, 1, 2$) and valleys in between are very clearly drawn. However, some minor shear bond failure (sbf) has been observed in the bolts by the numerical simulation. Further, the shear stress tends to zero close to midspan (at $x=2.5$) where the contact between the two neighbouring strata is lost.

In figure 4b the span is 10m and the shear stress distribution is shown for the lowest interlayer at 0.25m above the roof surface. Again mountains at bolts (at $x=0.5, 2.5, 3.5$) and valleys in between are clearly drawn. However, at the bolt at $x=1.5$ the mountain does not develop due to shear bond failure (sbf) between the bolt and the rock. Similarly to the right of the bolt at $x=4.5$, the contact is lost and the developing shear stress at the bolt region is zero or low.

In figure 4c the span is 20m and the shear stress distribution is shown for the lowest interlayer at 0.50m above the roof surface. Again mountains at the bolts (at $x=0.5+8.5$) and valleys in between are drawn, which may not easily be distinguished. Further, at the bolts at $x=0.5, 1.5, 2.5$, shear bond failure (sbf) has been observed during the numerical simulation. At the bolt at $x=9.5$, contact between layers is lost.

It may be observed that mountains and valleys are clearly distinguishable in the shear distribution for the cases shown in figures 4a and 4b, whereas the interlayer considered is relatively close to the bottom of the beams. In the case shown in figure 4c, the distribution is quite flat over a large part of it, whereas the interlayer considered is relatively further away from the beam bottom.
Normal stress at critical cross sections

In column 6 of table 1, the analytically evaluated (according to formulae provided by Sofianos and Kapenis, 1998) equivalent extreme fibre stress $\sigma_{rs}$ is given for each of the three beam spans considered to have single layer thickness $t$. In column 8, the analytically evaluated equivalent extreme fibre stress $\sigma_{rb}$ is given for each of the three beam spans considered to have intact rock thickness equal to the bolt length $l_b$. Respective numerical evaluations at the abutment and the midspan are given, below the analytical equivalent (mean) ones, within parentheses. The extreme fibre stress of the tested bolted beams is anticipated to lie in between; the largest stress should be for the untensioned bolts. In column 7, the numerically evaluated extreme fibre normal stress $\sigma_{hn}$ is given, separately for the abutment and the midspan, for the three spans considered, for zero pretension force; the evaluated stresses are almost identical for the three bolt patterns considered for each span. It may be observed that these stresses are lower than those of the pertinent beams with single stratum thickness.

In figures 5 the normal stress distribution along the two critical cross sections, i.e. the abutment and the midspan, for the previously mentioned bolt pattern and pretension cases, i.e. $s_b=1m$ and $P_c=50kN$, is drawn for the three tested span cases. Ordinate is the y distance measured along the cross section from the roof corner at the abutment. Abscissa is the normal stress acting at the section. A saw type line may be drawn which indicates individual rotation of each stratum and gain and loss of contact at some of the interlayers. At the abutment, reduction in stress or even loss of contact occurs in the upper area of each stratum, whereas increase in stress or even gain of contact occurs in the lower part; vice versa occurs at the midspan. However, it must be noticed that straight lines are drawn between the evaluated nodes along the critical cross sections, whereas in the actual beams these lines are curves for which the saw picture will be more crispy.

Figure 5a. Axial stress distribution at the two critical cross sections for $s=5m$, $s_b=1m$, $P_c=50kN$
In figure 5a the span is 5m and the distance between the nodes in the y direction is equal to the stratum thickness, i.e. 0.10m. Thus, a rough only estimate of the stress distribution within each stratum along the two cross sections is made. Further, it may be observed a large region of contact of the strata with the abutment or midspan joint, which is not the case for the intact rock beam; the larger value of the extreme stress at the abutment (1.85MPa, out of the figure) compared to that at midspan, conforms with the behaviour of the intact rock beam.

In figure 5b the span is 10m and the distance between the nodes in the y direction is equal to the half of the stratum thickness. Thus, an improved estimate of the stress distribution within each stratum along each of the two cross sections is made. A region of contact loss within each stratum may be seen here more clearly. Again, it may be observed a large region of contact between the strata at the abutment or midspan joint, and a larger value of the extreme stress at the abutment compared to that at midspan.

In figure 5c the span is 20m and each stratum has six nodes in the y direction along the critical cross sections. Thus, a an even better estimate of the stress distribution within each stratum along the two cross sections is made. A region of contact loss within each stratum may be seen here very clearly. Further, it may be observed the contact of almost all the strata with the abutment or midspan joint, and a larger value of the extreme stress at the abutment compared to that at midspan.
In figures 6 the maximum stress at or close to the extreme fibre of the critical sections is related to the applied bolt pretension pressure, at the critical cross sections of all tested beams. In figure 6a the pretension effect is related to the maximum normal stress at or close to the lowest extreme fibre stress of the abutment cross section, whereas in figure 6b the pretension is related to the maximum normal stress at or close to the uppermost fibre of the midspan cross section. From the trendlines given in the charts, it may be observed an only weakly and non clearly decreasing extreme stress with increasing pretension.

Conclusions

The behaviour of a selected range of stratified, cable bolted and pretensioned voussoir rock beams, belonging to an underground roof, is investigated numerically with the use of a discontinuum finite difference computer code. For some selected cases bolting is shown illustratively to allow for the development of shear stresses along interlayer surfaces, whereas shear bond failure of the bolts to cause an immediate reduction of the transmitted shear stress in its tributary region.

Non pretensioned cable bolts developed tension during deflection of the beams and caused decreased deflections and normal stresses at the extreme fibre of the two critical cross sections. However, pretensioning of the particular bolts was in most cases not very effective in reducing further substantially the deflections at midspan and the maximum normal stresses at the critical sections.

For a more clear understanding of the behaviour of the underground hard stratified rock voussoir beam roof, a wider range of cases is suggested to be studied. Further, alternative codes shall be applied to even denser mesh configurations and finally validated (Economopoulos et al., 1994) with in situ observations.
References


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