Development of an Integrated Pillar Strength Determination Based on Australian and South African Case Histories

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Abstract: A database of both failed and unfailed Australian underground coal mine pillar case studies was developed between 1991 and 1996 by the School of Mining Engineering at the University of New South Wales (UNSW) Australia. The original and more extensive South African coal pillar database used by Salamon and Munro in 1966 was also supplemented by that collated by Madden and Hardman in 1991/92 to give a combined database of 63 collapsed and 114 unfailed pillar systems. The Australian and South African databases were statistically analysed independently and as a combined data set using the maximum likelihood method. The earlier UNSW database of Galvin, Hocking and Salamon was updated and reanalysed as apart of this process, taking into account the effect of rectangular pillars on strength. It was found that there was less than a 4% variance in pillar design extraction ratios resulting from each of these approaches. The overall conclusion is that there is a remarkable consistency between the design formulae developed from back-analysis of the two separate national pillar databases containing many different coal seams and geological environments.

Key Words: rectangular pillars, pillar strength, pillar strength formulae, pillar collapse

1. INTRODUCTION

In the three year period to 1992, 60 continuous miners were trapped by falls of strata for more than seven hours in collieries in New South Wales. In the preceding two years, eight coal miners were killed in pillar extraction operations in the state. In the New South Wales and Queensland coalfields at least 15 extensive collapses of bord and pillar workings occurred unexpectedly in the 15 years to 1992. Six of these collapses occurred in working panels, fortuitously, five of which were during shut-down periods and the sixth while the continuous miner was being flitted to the surface for repairs.

Against this background, the Strata Control for Coal Mine Design Project (SCCMD) was established in the School of Mining Engineering at the University of New South Wales in late 1991. The project was funded by the coal industry insurers, namely the Joint Coal Board. Its primary objective was to improve the understanding of coal pillars and associated floor and roof strata under various loading conditions. The project aimed to develop practical design guidelines for bord and pillar workings and later longwall workings. A research team comprising a mix of both national and international expertise and of academic and industry experience and expertise was assembled for the project.
2. RESEARCH METHODOLOGY

The approach adopted to pillar design was based on that of Salamon and Munro (1966, 1967). Their study was unique because it was a first attempt to back calculate the in situ strength of a natural structural element such as an underground coal pillar from data representing experience in the field. Three decades later, the pillar strength formula resulting from this study and the pillar design method which evolved from it remain largely unchanged and in use in South Africa. The experience accumulated whilst using the strength formula to design tens of thousands of coal pillars in South Africa, together with a lack of another equally widely tested method of estimating the strength of coal pillars, encouraged the testing of the approach in Australia.

This paper is focussed solely on bord and pillar first workings because this is the only configuration where the pillar load can be computed without recourse to models of rockmass behaviour. Once the pillar strength formula has been quantified, it has wider application, for example in pillar extraction or in the design of chain pillars in longwall mining. Rockmass behaviour models may be required in these instances to estimate pillar load.

3. EMPIRICAL COAL PILLAR STRENGTH ESTIMATIONS

The development of computer and numerical technologies in recent decades has facilitated, at least in principle, the analysis of stresses in pillars and their foundations, that is, the roof and floor strata. Unfortunately, physical experimentation has not advanced equally rapidly.

Hence, our understanding of the intrinsic constitutive laws controlling the behaviour of yielding rocks is still unsatisfactory. More immediate problems include the significant discrepancies between the physical properties exhibited by rocks in situ and those measured in the laboratory by testing small specimens. These problems relate to the effects of size and shape on rock strength.

Many investigators have proposed simple empirical formulae to describe the strength of coal pillars. The most common feature of most of these empirical relationships is that they define strength ostensively only in terms of the linear dimensions of the pillars and a multiplying constant, representing the strength of the unit volume of coal. Investigators over the years have proposed formulae that belong to one of two types. One type defines pillar strength simply as a linear function of the width to height ratio:

\[
\sigma_{si} = K_I \left[ r + (1-r) \frac{w}{h} \right] \tag{1}
\]

where \( K_I \) is the compressive strength of a cube and \( r \) is a dimensionless constant. The quantities of \( w \) and \( h \) are width and height of the pillar respectively. If the cross-section of the pillar is rectangular, \( w \) usually refers to the smaller of the two side lengths. If the notation:

\[
R = \frac{w}{h} \tag{2}
\]
is introduced then Equation 1 becomes:

$$\sigma_{s1} = K_1(r + (1-r)R)$$  \hspace{1cm} (3)

According to this formula, geometrically similar pillars have the same strength regardless of their actual dimensions.

A second commonly used pillar strength formula takes the form of:

$$\sigma_{s2} = K_2\left(\frac{w}{w_0}\right)^\alpha\left(\frac{h}{h_0}\right)^\beta$$  \hspace{1cm} (4)

which is expressed in a dimensionally correct form. \(\alpha\) and \(\beta\) are dimensionless parameters, \(w\) and \(h\) are the linear dimensions of the pillar. Multiplier \(K_2\) is the strength of a reference body of coal of height \(h_0\) and a square cross-section with side length \(w_0\).

In most instances, the reference body is taken to be cube of unit volume for convenience sake, in which case \(h_0\) and \(w_0\) are both unity and can be omitted from the formula. Expressions belonging to this family are referred to as power law strength formulae. In contrast to formulae of the form of Equation 1, these formulae are also volume sensitive.

4. SOUTH AFRICAN PILLAR STRENGTH FORMULAE

In 1966, Salamon and Munro undertook a statistical back analysis of pillar performance in South Africa using the maximum likelihood method. Their field database comprised 27 failed cases and 98 unfailed cases. The researchers took care to ensure that a number of criteria were satisfied before a particular case was included in the database.

Firstly the area mined had to be sufficiently large to ensure that the tributary area estimate of the pillar load was an acceptably good approximation. Secondly, if pillar failure had occurred, it must not have happened immediately after the formation of the pillars. If such a case were included there would be some uncertainty with regard to the load that caused the pillars to collapse. Thirdly, the mining geometry in the area had to be reasonably uniform. Fourthly, if the case was to be included amongst the unfailed cases, it had to be ascertained that the time elapsed since mining was greater than a predetermined period. Fifthly, every effort was made to ascertain that a failure was, in fact, a pillar collapse (and not a failure in the roof or floor). Finally, the relevant dimensions of the layout (depth below surface, bord widths, pillar widths, working or pillar height and the smaller pillar angle if the pillars were not rectangular) was to be available.

The statistical analysis produced the following formula:

$$\sigma_{s2} = 7.2 \frac{w^{0.46}}{R^{0.56}}$$  \hspace{1cm} (MPa)  \hspace{1cm} (5)
In 1968 Bieniawski produced a linear formula to describe the strength of South African coal pillars. The formula was based on the testing of small scale "pillars" and took the form of:

$$\sigma_n = 6.2 \left( 0.064 + 0.36 \frac{w}{h} \right)^1 \text{ (MPa)}$$  \hspace{1cm} (6)

Since 1966, Salamon and Munro's formula has been used almost exclusively to design over 1.5 million pillars in South Africa. The field performance of the formula has corresponded closely with statistical predictions by Salamon and Munro.

5. EQUIVALENT WIDTH OF NON-SQUARE PILLARS

In practice, pillars are frequently non-square in plan. The load bearing capacity per unit area of the pillar with unequal side lengths is somewhat greater than that of a square pillar with the minimum width as its side length. This effect is due to the additional confinement that develops along the length of the pillar. In order to preserve the availability of the strength formulae, many have proposed the introduction of an equivalent width. The most promising recommendation came from Wagner (1974) who, making use of the concept of hydraulic radius, suggested that the effective width be defined as:

$$w_e = 4 \frac{A_p}{C_p}$$  \hspace{1cm} (7)

where $A_p$ and $C_p$ are the cross sectional area and the circumference of the pillar respectively. In the case of a pillar with a parallelogram shape (in plan view) sides $w_1$ and $w_2$ ($w_1 \leq w_2$) and an internal angle $\theta \leq 90^\circ$, Figure 1, the formula in (7) becomes:

$$w_{eo} = \Theta_o w$$  \hspace{1cm} (8)

where $w$ is the minimum width of the pillar, that is:

$$w = w_1 \sin \theta$$  \hspace{1cm} (9)

and the dimensionless factor $\Theta_o$ is defined by:

$$\Theta_o = \frac{2w_1}{w_1 + w_2}$$  \hspace{1cm} (10)

The range of this factor is $1 \leq \Theta_o < 2$, which is encountered as the aspect ratio moves from unity towards infinity. Experience indicates that well before the complete failure of a pillar, its edges are already yielding. Thus, if the width to height ratio of a pillar is low, one of the principal stresses confining its core will remain small and this stress, together with the maximum stress, will control failure. In such cases, the extra confinement that may arise from the aspect ratio will have little or no effect.
It is suggested that such apprehension may be catered for by postulating that the effective width is the minimum width, that is, \( w_e = w \) as long as \( R < R_t \) and it becomes \( w_e = w_{so} \) when \( R > R_u \). In the intermediate range, that is when \( R_t \leq R \leq R_u \), the effective width changes smoothly in accordance with:

\[
w_e = w_0 \Theta \left( \frac{R - R_t}{R_u - R_t} \right) \quad (11)
\]

Here the choice of the limiting \( w/h \) ratios is open to personal judgement. In the UNSW analysis, the values used are:

\[
R_t = 3 \quad R_u = 6 \quad (12)
\]

Using the concept of effective width, the power law in (4) can be rewritten for pillars that have a general parallelogram shape:

\[
\sigma_{zz} = K_2 w^\sigma h^\theta \Theta^\vartheta \quad (13)
\]

6. STRENGTH OF SQUAT PILLARS

Experience has shown that the Salamon and Munro formula (Equation 5) tends to underestimate the strength of pillars having a \( w/h \) ratio in excess of about 5. To cater for this problem, Salamon (1982) suggested an extension of Equation 4. This extension, referred to as the Salamon Squat Pillar Formula, after adaption to pillars of parallelogram shape, is as follows:
\[ \sigma_{\alpha} = K_{\alpha} V^{\alpha} R_{\alpha}^{\beta} \Theta^{\gamma} \left( \frac{R}{R} \right) - 1 + 1 \]  

(14)

where \( V = w^2 h \) and \( w \) is as defined in Equation (9).

Since its inception, this equation has been widely applied in South Africa using the following constants:

\[ R_{\alpha} = 5 \quad \alpha = 2.5 \]  

(15)

7. UNSW INITIAL DESIGN FORMULAE

Surprisingly, as at 1995 no distinct pillar design methodology existed in Australia. In Queensland, the dimensioning of coal pillars was at the discretion of the Mine Manager. In New South Wales, a new Coal Mines Regulation Act introduced in 1982 simply specified a pillar width of one tenth of depth with a minimum width of 10m.

In 1992, following a number of serious incidents related to there being no restriction on pillar height, the Chief Inspector of Coal Mines in New South Wales required operators to obtain approval to mine at heights exceeding 4m. To address the need for a pillar design methodology the SCCMD research team undertook in 1995 a preliminary analysis of its database (Hocking et al, 1995).

The database comprised 14 collapsed cases and 16 stable cases which satisfied the selection criteria of Salamon and Munro (Section 4). The analysis employed the same maximum likelihood statistical approach as that of Salamon and Munro (1966). Galvin and Hebblewhite (1995) subsequently published the following pillar design formulae which find application in Australia today.

\[ \sigma_{\alpha} = 7.4 \frac{w^{0.46}}{R^{0.66}} \text{ [MPa]} \]  

(16)

and its squat pillar version (\( R > 5 \)):

\[ \sigma_{\alpha} = \frac{1924}{w^{0.45} R^{0.25}} \left[ 0.237 \left( \frac{w}{5h} \right)^{25} - 1 \right] + 1 \]  

(MPa)  

(17)

Since minimum width was proposed as the effective width, it follows that \( \Theta = 1 \) in these expressions. The formula for strength based on the linear relationship takes the form:

\[ \sigma_{\alpha} = 5.36(0.64 + 0.36R) \]  

(MPa)  

(18)

These formula are plotted in Figure 2. Statistical analysis employing the maximum likelihood method revealed a marginally higher level of confidence when the \( K_{\alpha}, K_{\beta}, r, \alpha \) and \( \beta \)
parameters were all allowed to float. However, this resulted in a only a minor difference in pillar strength compared to when these parameters were fixed at $\alpha = 0.46$, $\beta = 0.66$ as per Salamon and Munro (1966) and $r = 0.64$ as per Bieniawski (1968).

![Graph showing average pillar stress vs. width/height ratio](image)

Figure 2 UNSW Pillar Strength Formulae Outcomes - 1995.

Given the starting base of the Australian coal industry regarding pillar design formulae and in order to minimise confusion, Galvin and Hebblewhite decided to retain for the $\alpha$, $\beta$ and $r$ parameters, those values associated with Salamon and Munro’s Power Formula, Salamon’s Squat Pillar Formula and Bieniawski’s Linear Formula.

For similar reasons, statistical analysis presented to the industry was based on using the minimum pillar width in the formulae. Nevertheless, a range of statistical analysis was undertaken using both minimum pillar width, effective pillar width and floating parameters. In all cases, the power law form of the pillar strength equation yielded a lower standard deviation (higher level of confidence) than the linear form.

The use of factor of safety (FOS) is a means of incorporating acceptable and tolerable levels of risk into engineering designs.

$$\text{Risk} = \text{Probability of failure} \times \text{Consequence}$$

When the consequences of failure are serious, a reduced probability of failure needs to be adopted in order to achieve an acceptable level of risk. For coal pillars:

$$\text{Factor of Safety} = \frac{\text{Pillar Strength}}{\text{Pillar Load}}$$
The initial probabilistic analysis of field performance data from Australian mines defined the following relationship between probability of failure and factor of safety (FOS). The table lists the appropriate FOS to be used with either of the UNSW formulæ in order to achieve the required Probability of Failure, consistent with an acceptable or tolerable level of risk for pillar design. The latest analysis of the current database results in minimal change in these risk values.

<table>
<thead>
<tr>
<th>Probability of Failure</th>
<th>FOS (Linear)</th>
<th>EOS (Power)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 in 10</td>
<td>0.84</td>
<td>0.87</td>
</tr>
<tr>
<td>5 in 10</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>1 in 10</td>
<td>1.29</td>
<td>1.24</td>
</tr>
<tr>
<td>1 in 100</td>
<td>1.59</td>
<td>1.48</td>
</tr>
<tr>
<td>3 in 1,000</td>
<td>1.72</td>
<td>1.59</td>
</tr>
<tr>
<td>1 in 1,000</td>
<td>1.85</td>
<td>1.69</td>
</tr>
<tr>
<td>1 in 10,000</td>
<td>2.09</td>
<td>1.88</td>
</tr>
<tr>
<td>1 in 100,000</td>
<td>2.33</td>
<td>2.06</td>
</tr>
<tr>
<td>1 in 1,000,000</td>
<td>2.57</td>
<td>2.23</td>
</tr>
</tbody>
</table>

In some instances, the two forms of pillar strength formulæ generate nearly identical values. Nevertheless, the table of Probability of Failure and Factor of Safety indicates that a higher risk is assigned to the use of the linear formulæ. This apparent anomaly is a consequence of the greater scatter of data (hence, higher standard deviation) associated with the linear formulæ.

8. UPDATED AUSTRALIAN PILLAR STRENGTH FORMULÆ

By 1996, the Australian data set had expanded to include 19 collapsed cases and 16 unfailed cases. Salomon (Salomon et al 1996) subjected the data to a series of statistical analysis that was more comprehensive than that employed in the earlier investigations by Salomon and Munro (1966), Hocking et al (1995) and Galvin and Hebblewhite (1995). The effective pillar width procedure described earlier was applied and all parameters were allowed to float. The analysis revealed that the following power strength formulæ best describes the observed behaviour of coal pillars in New South Wales and Queensland:

\[
\sigma_z = 8.60 \left( \frac{w \Theta}{h^{0.25}} \right)^{0.51} \quad (\text{MPa}) \quad (19)
\]

The corresponding expression for squat pillars \((R > 5)\) is given by:

\[
\sigma_z = \frac{27.63 \Theta^{0.51}}{w^{0.22} h^{1.10}} \left[ 0.29 \left( \frac{w}{h} \right)^{2.5} - 1 \right] + 1 \quad (20)
\]

Figure 3 shows a comparison between the outcomes of these formulæ and that of the initial UNSW formulæ (Equations 16 and 17). In the case of a mining height of 2m, the figure shows that for a given pillar strength, pillars designed with the updated formulæ may be some
2.5m less in width. Whilst this may appear significant, for a bord width of 6m at a width to height ratio w/h of 10, this results in less than an extra 4% resource recovery. For similar circumstances in a 4m mining height environment, the reduction in pillar size is of the order of 2m.

The reduction in pillar size resulting from the application of the updated formulae is partially due to the expanded database and partially to the application of the effective pillar width formula, Equation 11. The Australian database contains a relatively high proportion of non square pillars and hence the application of the effective width concept could be expected to result in a increase in predicted pillar strength. Until more experience is gained with the concept, it is proposed by UNSW to err on the side of caution in Australia and to continue to base pillar strength calculations on the minimum pillar dimension.

![Graphs showing pillar strength vs width/height ratio for different heights](image)

(a) $h = 2\text{m}$  
(b) $h = 4\text{m}$

Figure 3: Comparison between UNSW Power Formulae - 1995 and 1996 - for 2m and 4m pillar heights

9. RE-ANALYSIS OF SOUTH AFRICAN DATABASE.

The original extensive South African coal pillar database used by Salamon and Munro in 1966 has since been updated and supplemented by Madden and Hardman (1992). This combined South African database comprises 44 failed cases and 98 unfailed cases. It has also been re-analysed using the same statistical techniques as for the Australian database. Two failed cases were later omitted from the data set (see Salamon et al 1996).

This analysis has produced the following strength formulae.
\[ \sigma_{12} = 6.88 \left( \frac{w \Theta}{h^{0.6}} \right)^{0.42} \text{ (MPa)} \]  
(21)

The corresponding expression for squat pillars \((R > 5)\) is given by the expression:

\[ \sigma_{12} = \frac{16.36 \Theta^{0.42}}{w^{0.116} h^{0.016}} \left\{ 0.215 \left[ \left( \frac{w}{h} \right)^{23} - 1 \right] + 1 \right\} \text{ (MPa)} \]  
(22)

The linear version of the strength estimator is simply:

\[ \sigma_{l1} = 5.60 (0.69 + 0.31 R) \text{ (MPa)} \]  
(23)

Figure 4 shows the comparison between the pillar strength produced by Equations 5 and 6 and Equations 21 and 22. In the case of a mining height of 2m, the figure shows that for a given pillar strength, pillars designed with the updated formulae may need to be some 2m more in width. For a bord width of 6m at a width to height ratio of 10, this results in about 3% less resource recovery. For similar circumstances in a 4m mining height environment, the increase in pillar size is of the order of 3.2m.

10. COMBINED AUSTRALIAN AND SOUTH AFRICAN DATABASES

A further step in the research program was to combine the South African and Australian databases and to analyse them as a combined population and then compare and contrast them with the two independent data populations for each country.

This combined database comprised 177 cases of pillar systems including 61 collapsed cases. This produced the following formulae:

\[ \sigma_{12} = 6.88 \sqrt{\frac{w \Theta}{h^{0.6}}} \text{ (MPa)} \]  
(24)

For \(R > 5\), the squat version of this expression takes the form:

\[ \sigma_{12} = \frac{19.06 \sqrt{\Theta}}{w^{0.116} h^{0.016}} \left\{ 0.253 \left[ \left( \frac{w}{h} \right)^{23} - 1 \right] + 1 \right\} \text{ (MPa)} \]  
(25)

The corresponding linear formula can be expressed simply as:

\[ \sigma_{l1} = 5.41 (0.63 + 0.37 R) \text{ (MPa)} \]  
(26)

Figure 5 shows failed and unfailed cases in the load plane. The figure illustrates a fairly good discrimination between the two sets of points. Only one unfailed point occurs on the wrong side of the \(s = 1\) line and the median failed cases is 1.039.
Figure 4 Comparison between South African Power Formulae - 1966/82 and 1996.

Figure 5 The failed (o) and unfailed (+) cases in a pillar strength versus load plot for the combined Australian and South African data bases.

Figures 6(a) and (b) shows a comparison between pillar strengths using power law estimators derived from the Australian, South African and the combined Australian and South African databases. The closeness of the predictions is remarkable considering the geographical separation of the Australian and South African coalfields.

11. CONCLUSIONS

One of the main purposes of the SCCMD project was to develop a pillar design formula that can be applied to Australian conditions. This has been accomplished relatively early in the project.
Figure 6 Comparison between power law strength formulae derived for Australian, South African and combined data bases - pillar heights 2m and 4m.

In order to enhance confidence in the pillar design method, additional research was undertaken. It was noted that the formula derived from the initial Australian data base resembled closely the original Salamon and Munro expression. This somewhat surprising resemblance prompted further research and the enlargement of the data base. The larger data base yielded pillar strengths that again were similar to those obtained from the initial UNSW research and by Salamon and Munro. The combination of the Australian and South African data bases reinforced the original impression, namely that the underlying pillar strengths in these countries resembled each other closely.

The outcome of the investigation lends support to the view expressed by Mark and Barton (1996). They suggested that strength values obtained in the laboratory cannot be utilised in a meaningful way in pillar design and the variation in the strength of pillars of the same size can be disregarded in many instances. Other investigators have come close to making a similar statement (Mark, 1990, Salamon, 1991, Galvin et al, 1995).

Mark and Barton emphasise that they do not claim that the in situ strength of all US coal is the same. Their study merely showed that a uniform strength is a better approximation than one based on laboratory testing.

Whilst the UNSW research conclusions are encouraging, complacency is not justified. The formulae are based on competent roof and floor conditions. Significantly different pillar strengths may be associated with abnormal strata behaviour mechanisms.
Since pillars with \( w/h \) ratios greater than 10 have not been tested to destruction, it must also be recognised that neither linear nor power law formulae have been validated at \( w/h \) ratios greater than about 8.

REFERENCES


