A NEW METHOD FOR COAL PILLAR DESIGN

By

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ABSTRACT

A failure criterion for coal is proposed in which a distinction is made between brittle and "pseudo-ductile" failure. This criterion is then used in the development of new equations for coal pillar strength, including a transition from brittle fracture to pseudo-ductile yielding. In addition, equations are obtained which allow the critical minimum pillar dimensions to be calculated to avoid "catastrophic" and "ultimate failure".

The use of these formulae is then demonstrated by comparing the results with pillar case histories derived from the literature.

INTRODUCTION

A key consideration in the design of coal pillars is the choice of the pillar size that will allow maximum coal recovery while maintaining overall stability. Wilson (1972) first proposed the concept that a pillar could be regarded as comprising a central, solid core surrounded by a peripheral zone of fractured coal. However, use of Wilson's formulae in western Canada predicted fracture zones much less than those experienced in practice. Consequently, work was commenced to modify these ideas to better fit Canadian conditions, (Barron, 1983, 1994). The equations developed in this paper represent a further step in this work.

COAL FAILURE CRITERION

At high confining pressure the frictional resistance to sliding exceeds the shear resistance to fracture, thus it is easier to form a new fracture surface than to produce displacement along existing fractures.

At an intermediate confining pressure the shear resistance to fracture just equals the frictional resistance to sliding on the fracture surface; there is no drop from peak to the residual stress level and large deformations can take place with little or no increase in axial stress. Such deformation is called ductile when associated with metals in which there is no loss of cohesion. However, as explained above, similar deformation behaviour, with a loss of cohesion, can be caused by brittle fracture. The author calls this mechanism "pseudo-ductile" failure to distinguish it from pure plastic behaviour.

This failure mechanism has previously been invoked for hard rocks at great depth (Oravin, 1960; Maurer, 1985; Byerlee, 1967). It is suggested that such failure can occur in coal in the range of confining pressures normally encountered at coal mining depths.

The point of transition between brittle fracture and "pseudo-ductile" yield is given by the point of intersection of the curves of peak and residual stresses plotted against confining pressure. Such
an intersection can only occur if the peak stress curve is non linear. It is assumed that this curve is given by Hoek and Brown's (1982) expression:

\[ \sigma_{1p} = \sigma_{3p} + (m \sigma_{3p} + s \sigma_{e})^{1/2} \]  

(1)

where \( \sigma_{1p} \) and \( \sigma_{3p} \) are the major and minor stresses at peak failure, \( \sigma_e \) is the intact uniaxial compressive strength, \( m \) and \( s \) are constants for the coal and, for intact coal, \( s=1 \).

The residual stresses are assumed to obey Coulomb-Navier theory and are related by:

\[ \sigma_{1n} = P_B + k \sigma_{3n} \]  

(2)

where \( P_B \) is the residual uniaxial compressive strength, \( k \) is a constant for the coal and is given by:

\[ k = \frac{1 + \sin \phi_B}{1 - \sin \phi_B} \]  

(3)

where \( \phi_B \) is the angle of friction for broken coal.

The transition from brittle to pseudo-ductile failure occurs at the intersection of these two curves and is given by putting \( P_{3T} = \sigma_{3n} = \sigma_{3p} \) and \( P_{3p} = \sigma_{3p} = \sigma_{3p} \) in the above equations, this gives:

\[ P_{3T} = \frac{\beta \cdot \sigma_B \cdot (k-1)^2 \cdot (P_B' - \sigma_e)^{1/2}}{2(k-1)^2} \]  

(4)

and

\[ P_{3T} = P_B + k \cdot P_{3T} \]  

(5)

where

\[ \beta = \frac{2(k-1)P_B'}{m \sigma_e} \]  

(6)

Figure 2 shows the application of these equations to triaxial tests results from Pittsburgh coal reported by Kipanov (1981). The following parameters were derived from the analysis of this data:

\[ m = 13.9, \sigma_e = 21.3 \text{ MPa}, \text{ correlation coefficient } r = 0.98 \]

\[ P_B = 10.5 \text{ MPa}, k = 3.25, \text{ and } \phi_B = 31.9^\circ \]

(7)

and the transition point is given by:

\[ P_{3T} = 17.5 \text{ MPa}, P_{3T} = 50.7 \text{ MPa} \]

Unfortunately these tests were terminated at a maximum confining pressure of 34 MPa, below that estimated above for the onset of pseudo-ductile yield. Nevertheless, it does not take much extrapolation of the results to obtain the transition point. This is taken as strong indirect evidence that the proposed failure criterion applies to coal. It is recommended that triaxial tests on coal should be carried out at considerably higher confining pressures than is current practice.

**PERIPHERAL FRACTURE ZONE AROUND A PILLAR**

Consider an elemental slice of width \( dx \) at a depth \( x \) in the fractured portion of the periphery of a coal pillar, as shown in figure 3.

\[ \mathbf{F} = c_B + P_1 \tan \phi_B \]

Fig 3: Forces acting on a strip in the fractured pillar periphery.

Let the pillar have a height \( H = 2h \) and a width \( B = 2b \), let \( t \) be the shear stress across the ends of the element, \( c_B \) be the cohesion and \( \phi_B \) be the angle of friction of the failed coal. Let \( P_1 \) be the vertical stress acting on this element with \( P_2 \) and \( (P_2 + dP_2) \) the confining pressures on either side of the element. Then, ignoring the weight of the element, for equilibrium:

\[ H \cdot dP_2 = 2(c_B + P_1 \tan \phi_B) \cdot dx \]  

(7)

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Further, if we assume that the broken coal strength is defined by equation 2, then substituting into equation 7 and integrating between the limits \( x = 0 \), \( P_1 = P_0 \) and \( x = x, P_1 = P_1 \), the following expressions are obtained:

\[
P_1 = P_0 \left( e^{\frac{k(x) \tan \theta_B}{k}} \right) \left( \frac{1}{k} \right)
\]

(8)

where \( \alpha = \frac{(1 + \sin \theta_B)}{2 \sin \theta_B} \) and \( H = 2h \)

and

\[
P_3 = \left( \frac{P_1 - P_3}{P_0} \right) \left( \frac{1}{k} \right)
\]

(9)

i.e. these equations describe the build-up of the vertical and horizontal stresses in the fractured zone until the transition point is reached when \( P_1 = P_1T \) and \( P_3 = P_3T \).

Beyond the transition point the slice yields, and it is assumed that the vertical stress remains a constant at \( P_1T \); it follows that in this yield zone \( P_3 \) also remains constant at \( P_3T \).

If the transition point occurs at a depth \( d_T \) from the pillar edge, then \( d_T \) can be determined from equation 8 by putting \( x = d_T \) and \( P_1 = P_1T \) and solving for \( d_T \):

\[
d_T = \frac{h \tan \theta_B \left[ \log_e \left( \frac{P_1T}{P_0} \right) + \frac{1}{k} \right]}{k}
\]

(10)

**PILLAR STRENGTH FORMULAE**

Let \( \sigma_p \) be the mean stress applied to a pillar of width \( B (=2b) \) and height \( H (=2h) \). Let \( S_B \) be the vertical field stress and \( Y = (\sigma_p / S_B) \) is the mean pillar stress concentration factor. Let \( \sigma_C \) be the pillar strength, i.e. the mean peak stress at which failure first reaches the pillar pole.

Assuming symmetry, consider only the stress distribution across half the pillar at its mid height. Prior to fracture initiation the vertical stress distribution is similar to that shown in figure 4(a).

The maximum vertical stress at the mid height of the pillar is that on the boundary, hence for fracture to initiate, this stress must be equal to the uniaxial compressive strength, \( \sigma_c \), of the intact coal. This condition will occur when:

\[
\sigma_1 = \frac{Y}{Y_m} \sigma_c
\]

(11)

where \( \sigma_1 \) is the mean pillar stress for fracture initiation at the boundary and \( Y_m \) is the stress concentration factor at the mid height of the pillar boundary. \( Y_m \) can be obtained from the work of Obert and DuVal (1967) for pillars with adjacent rooms of different shapes.

Once fracture initiates, the fracture zone will continue to grow until the confining pressure is sufficient to prevent further fracturing. The stress distribution will then be as illustrated in figures 4b-1 and 4b-2, in which there is a peripheral fracture (or fracture plus yield) zone with a central elastic core. In this case the area under the stress distribution curve will be equal to that under the mean pillar stress line and equilibrium will prevail.

**NARROW PILLARS**

The confining pressure in the broken coal builds up exponentially (equation 8). For narrow pillars, once fracture has initiated, the confining pressure build up is insufficient to prevent fracture propagating to the centre of the pillar. Hence failure propagates immediately to the pillar centre and the strength of the pillar must be equal to the mean pillar stress required to initiate fracture:

\[
\sigma_1 = \sigma_{C,mm} = \frac{Y}{Y_m} \sigma_c
\]

(12)

i.e. this is the minimum pillar strength. In this case, the area under the stress distribution curve (figures 4c-1 and 4c-2) is less than that under the mean stress line; this unstable situation has been called "catastrophic" failure (although whether or not this failure is violent will depend on the relative stiffnesses of the fractured pillar and the mine).

There will be some critical semi-pillar width, \( b_c \), beyond which such catastrophic failure cannot occur, this is given by equating the minimum pillar strength with the area under the stress distribution curve:

when \( \sigma_1 > \sigma_c \):

\[
\frac{Y}{Y_m} \sigma_c b_c = \int_0^{b_c} P_1 dx
\]

(13)

\[
= \frac{P_0 \left( h \tan \theta_B \left( e^{k(b_c / h) \tan \theta_B - 1} - 1 \right) \right)}{2 \sin \theta_B}
\]

where \( \theta = \frac{(1 - \sin \theta_B)}{2 \sin \theta_B} \)

and when \( \sigma_1 < \sigma_c \):

\[
\frac{Y}{Y_m} \sigma_c b_c = (b_c + d_T) P_1T + \int_0^{d_T} P_1 dx
\]

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Fig 4: Modes of pillar stability and failure

\[ \sigma_p = \sigma_{SF} \]

\( (b_c - d_c) P_T \) \[ \sigma_{SF} = \frac{h}{b} \left( \frac{k(b/h) \tan \theta_B - 1}{\tan \theta_B} \right) \cdot (d_c) \] \[ \sigma_{SF} = \frac{h}{b} \left( \frac{k(b/h) \tan \theta_B - 1}{\tan \theta_B} \right) \cdot b \] \[ \gamma S_y = \frac{P_T}{b_c} \left( \frac{h}{b} \right) \left( \frac{k(b/h) \tan \theta_B - 1}{\tan \theta_B} \right) \cdot b \] \[ \gamma S_y = \frac{P_T}{b_c} \left( \frac{h}{b} \right) \left( \frac{k(b/h) \tan \theta_B - 1}{\tan \theta_B} \right) \cdot b \]

"Ultimate" failure. It can be shown that the pillar strength in this case is given by:

When \( d_c > b \), the ultimate fracture strength, \( \sigma_{SF} \), is given by:

and the critical pillar dimension, \( b_c \), to avoid ultimate fracture can be obtained by solving the following equation for \( b_c \):

These equations can be solved by successive approximation to give the critical pillar dimensions to avoid catastrophic failure.

**Wide Pillars**

When the pillar width is greater than this minimum, the fracture will propagate and stop, leaving the pillar with a central elastic core. As the load on the pillar continues to increase, the failed zone will eventually reach the centre and the pillar is deemed to have failed. In this case, the area under the stress distribution curve just equals the area under the mean stress line. This is called

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\[ \sigma_{ST} = \left(\frac{b - d_T}{b}\right) P_{TT} \]

and the critical pillar dimension to avoid ultimate yield, \(b_{u} \), is obtained by solving equation 18 for \(b_{u} \):

\[ Y_{SV} = \left(\frac{b_{u} - d_T}{b_{u}}\right) P_{TT} \]

SOLID AND BROKEN COAL PROPERTIES

To use the above equations four basic coal properties are required, namely: \( P_{B} \) and \( k \) (or \( \theta_{G} \)) for broken coal and \( \sigma_{c} \) and \( m \) for intact coal.

BROKEN COAL PROPERTIES

It is believed that a good value of \( \theta_{B} \), and hence \( k \) can be obtained from the slope of the residual stress curve. However, the value of \( P_{B} \) obtained from this curve is probably an overestimate. It is suggested that a small but finite value of \( P_{B} \) be assumed, perhaps 0.1 times the value of \( K_{S} \) given below or, better still, it be derived from a back analysis of the depth of the fracture zone measured in a pillar in the field. (Note that this value can have a significant influence on the magnitude of the pillar strength calculated from the above equations.)

It is also assumed that the broken coal properties are uninfluenced by size effects.

INTACT COAL PROPERTIES

Now \( \sigma_{c} \) and \( m \) can be determined from triaxial tests on laboratory size specimens. However, it is well known that the intact coal strength is exceedingly sensitive to the specimen size, hence consideration must be given to how these values must be modified to account for size effects.

It is assumed that the laboratory tests are carried out on cylindrical specimens of height twice the diameter and of a diameter of the order of 5 cm. Now the value of the uniaxial compressive strength obtained from these tests, \( \sigma_{c} \), can be related to the uniaxial curve strength, \( K_{c} \), by the expression (Barron, 1963):

\[ \sigma_{c} = K_{c} \left(\frac{D_{s}}{H_{s}}\right)^{1/2} \]

and when \( (D_{s})^{1/2}/H_{s} > 0.784 \):

\[ \sigma_{c} = 0.784 K_{s} \]

where \( D_{s} \) and \( H_{s} \) are the specimen width and height respectively (in metres) and \( K_{s} \) is numerically (but not dimensionally) equal to the 1-metre cube strength. For most laboratory size specimens equation 19 applies and \( K_{s} \) can be determined.

Following the examples quoted by Hustrulid (1976), it is believed that the appropriate uniaxial compressive strength, \( \sigma_{c} \), for use in the pillar equations is that for a cube of height equal to the pillar height. For most mine pillar dimensions this means that equation 20 applies. Hence replacing \( \sigma_{c} \) in equation 1 by that from equation 20 gives:

\[ \sigma_{IP} = \sigma_{TP} + \left( m K_{S} K_{IP} s^{2} \right)^{1/2} \]

where \( m = 0.784 \) and \( s = 0.616 \) for intact coal.

For example, using the Pittsburgh coal data reported above, assuming that the laboratory specimens were 10 cm. high by 5 cm. diameter gives:

\[ \sigma_{IP} = \sigma_{TP} + \left( 10.9 \sigma_{c} K_{S} + 0.516 K_{S}^{2} \right)^{1/2} \]

i.e. \( \sigma_{IP} = 7.47 \) MPa, \( K_{S} = 9.52 \), \( P_{B} = 0.95 \) MPa, \( P_{TT} = 66.5 \) MPa and \( F_{TT} = 20.2 \) MPa.

CASE HISTORIES

The applicability of these equations will now be tested by comparison with case histories reported in the literature.

PILLAR STRENGTH FORMULAE

Salamon and Munro (1967) derived an empirical equation for the strength of South African coal pillars as follows:

\[ \sigma_{c} = 7.09 \left(\frac{D}{H^{0.60}}\right) \left(\frac{H}{1.5 H^{0.60}}\right) \]

Bieniawski (1968), from large scale in-situ testing of South African coal, derived the following pillar strength formula:

\[ \sigma_{c} = 2.72 \times 1.5 \left(\frac{B}{H^{0.50}}\right) \]

where \( B \) is the pillar width and \( H \) is its height in metres and \( \sigma_{c} \) is in MPa.

The strength variation for a 3 m. high pillar for a range of (B/H) ratios from 0.5 to 6.0 have been calculated using each of these formulae. To use the equations developed in this paper, the following properties were assumed: \( K_{S} = 7 \) MPa.
$P_b = 0.7 \text{ MPa}$ and $\theta_h = 27^\circ$, the value of $m'$ was then varied to give an approximate best fit to the other results, this occurred when $m' = 1.96$.

Figure 5 shows the results of this comparison. The shape of the curve from the new formula is completely different from that of the others. In particular, the new formula predict that there is a minimum pillar width below which the pillar strength remains constant. This appears to the author to be more realistic than the implication of the other equations that pillar strength continually diminishes with decreasing width.

![Figure 5: Comparison of pillar strength formulae](image)

However, both the previous equations are empirically derived and thus give a reasonable fit to South African experience. It was therefore decided to treat the results calculated from the new formulae as if they were field data points and then to correlate them with $\frac{(B+H)}{H}$ in the form firstly, of Soliman and Munro and secondly, of Bieniawski. This yielded the following equations:

$$\sigma_s = 7.47 \left( \frac{0.806}{(B+H)} \right), r = 0.96$$

and

$$\sigma_b = 3.1 \times 1.5 \left( \frac{B}{H} \right), r = 0.95$$

When treated in this manner there is a surprisingly good agreement with the empirically derived formulae, with correlation coefficients no worse than were obtained when the empirical formulae were developed.

Thus it may be concluded that the new equations give results compatible with these previous studies. In addition, because allowance has been made for pseudo-ductile yielding to occur, the new formulae should not be limited by some upper value of $\frac{(B+H)}{H}$ to which they are applicable.

**A YIELDING PILLAR**

Skelly et al. (1977) loaded a test pillar to failure by the sequential reduction of its' cross section from an initial 24.4 m. square through steps of 18.9, 13.4, 11.6, 9.8, and 7.9 m. square. They measured the mean pillar stress and the deformation changes that took place: the solid line in figure 6 gives their results.

From laboratory data given by these workers, $K_N$ was estimated to be 2.6 MPa. It was also assumed that $\theta_h = 27^\circ$ and that $m'$ was the same as for Pittsburgh coal = 10.9. The value of $P_b$ was varied to obtain the best approximate fit to their results: this occurred with $P_b = 0.15$ MPa. The pillar strengths calculated from the new equations using this data are shown as the points on figure 6.

The agreement between these calculated results and the measured data is good. This is probably the strongest evidence to date that the mode of pillar failure is one of pseudo-ductile yield.

![Figure 6: A yielding coal pillar](image)

**CONCLUSIONS**

1. The pillar strength formulae developed here take into account the possibilities of the pillar failure occurring by brittle fracture alone or by a combination of brittle fracture and pseudo-ductile yield. As a result, unlike most other approaches, there is not an upper limit for which they are applicable.
2. The equations also predict that there must be some lower limit to the pillar strength and that it does not continuously decrease with reduction of pillar width.

3. Equations have also been derived which allow the critical pillar dimensions to avoid catastrophic fracture or yield and ultimate fracture or yield to be determined. This should be useful for design purposes.

4. Comparison of the results with case histories indicates that the formulae can satisfactorily be made to fit the experimental data. However, since inevitably such case histories do not include all the required parameters, assumptions had to be made for some of the coal properties. Hence this approach cannot be said to have been proven, although there is indirect evidence to suggest that it is close to the truth and, in particular, that the transition from brittle to ductile failure occurs in coal pillars at the stress levels normally encountered in coal mining.

5. An attempt has been made to include the well known effects of size on coal strength in this development. However, it is suggested that more work is required in this area, particularly with respect to the effects of size on coal strength when under triaxial stress conditions.

ACKNOWLEDGEMENTS

The author would like to acknowledge the financial assistance received from CANDM, Dept. Energy, Mines and Resources, Ottawa and from the National Council for Scientific and Engineering Research towards this work.

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The AusIMM Illawarra Branch, Ground Movement and Control related to Coal Mining Symposium August 1986

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