SUBSIDENCE AND STRAIN MEASUREMENTS

Subsidence and strain measurements of a
full grid established on the surface have
been carried out during and after the
extraction of each of the four longwalls.
Grid lines are shown in Fig.2.

Subsidence and strain profiles, as
measured over the longwalls, are given in
Fig.3 and maximum values are given in Table
2.

Subsidence due to Longwall I was quite
small, but increased rapidly with the
extraction of the second and subsequent
longwalls.

The measured strains did not fit the
classical pattern as observed in Europe,
but rather demonstrated the new familiar
irregularities of the Southern coalfield.

TABLE 2

<table>
<thead>
<tr>
<th>EXTRATION</th>
<th>MAXIMUM</th>
<th>MAXIMUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>LONGWALL</td>
<td>SUBSIDENCE, m</td>
<td>STRAIN, mm/m</td>
</tr>
<tr>
<td>LINE F</td>
<td>LINE F</td>
<td>TENSILE COMPRESSION</td>
</tr>
<tr>
<td>I</td>
<td>0.095</td>
<td>0.090</td>
</tr>
<tr>
<td>I+II</td>
<td>0.541</td>
<td>0.513</td>
</tr>
<tr>
<td>I+II+III</td>
<td>0.760</td>
<td>0.754</td>
</tr>
<tr>
<td>I+II+III+IV</td>
<td>0.800</td>
<td>0.800</td>
</tr>
</tbody>
</table>

The most significant irregularities
occurred at areas of predominant jointing
and high topographic variation.

Table 3 gives the ratio of maximum
subsidence to seam thickness and the ratio
of longwall width (including pillar width
between two longwalls) to mining depth.

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Fig. 2 - West Cliff Colliery Subsidence Investigation Over No.3 Area Longwalls

<table>
<thead>
<tr>
<th>LONGMALL NO.</th>
<th>I</th>
<th>I + II</th>
<th>I + II + III</th>
<th>I + II + III + IV</th>
<th>I + II + III + IV + V</th>
</tr>
</thead>
<tbody>
<tr>
<td>WIDTH W (m)</td>
<td>143</td>
<td>335</td>
<td>529</td>
<td>719</td>
<td>911</td>
</tr>
<tr>
<td>W/h</td>
<td>0.71</td>
<td>0.72</td>
<td>0.74</td>
<td>1.14</td>
<td>1.55</td>
</tr>
<tr>
<td>SEAM THICKNESS (m)</td>
<td>2.8</td>
<td>2.8</td>
<td>2.7</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>LINE E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAXIMUM SUBSIDENCE (m)</td>
<td>0.095</td>
<td>0.541</td>
<td>0.700</td>
<td>0.791</td>
<td>0.800</td>
</tr>
<tr>
<td>E/m</td>
<td>0.030</td>
<td>0.193</td>
<td>0.259</td>
<td>0.283</td>
<td></td>
</tr>
<tr>
<td>LINE F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAXIMUM SUBSIDENCE (m)</td>
<td>0.090</td>
<td>0.513</td>
<td>0.754</td>
<td>0.850</td>
<td>0.880</td>
</tr>
<tr>
<td>E/m</td>
<td>0.036</td>
<td>0.166</td>
<td>0.279</td>
<td>0.304</td>
<td></td>
</tr>
<tr>
<td>PREDICTED MAXIMUM SUBSIDENCE</td>
<td>0.126</td>
<td>0.556</td>
<td>0.760</td>
<td>0.924</td>
<td>0.970</td>
</tr>
</tbody>
</table>

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R.D. Lana, P. Moxon, D.M. Shu

**PREDICTION OF MAXIMUM SUBSIDENCE**

Based on the derived relationship, the maximum subsidence values of 0.1m and 0.18m are estimated along Line E and Line F respectively, during the extraction of Longwall V. The critical width will have been reached after the extraction of L.W. V.

Subsidence data for full extraction for Australian conditions are given in Fig.5, and Fig.6 gives the correction factors for reduction of maximum subsidence due to the chain pillars as obtained from local measurements. This information was used to predict mining subsidence for Longwalls I to IV at West Cliff Colliery.

The method of calculation is given below:

- Extraction depth \( h = 405n \)
- Width of a single Lk \( W = 14m \)
- Width of a pillar \( Wp = 49m \)
- Seam thickness \( = 2.5 - 2.6m \)

**Longwall I**

\[
\frac{W}{h} = \frac{0.1}{0.3} \\
S/m = \frac{0.045}{0.1} \quad \text{(From Fig.5)} \\
S = 2.6 \times 0.045 = 0.126m
\]

**Longwall I + II**

Total width \( = 14m + 49m + 14m \)

\[
\frac{W}{h} = \frac{0.1}{0.72} \\
S/m = \frac{0.3}{0.1} \quad \text{(From Fig.5)} \\
S = 2.6 \times 0.3 = 0.10m
\]

Using Fig.6, a correction to \( S_1 \) is now made by taking a value of 46m as the effective width of the intervening pillars.

\[
Wp = 46m \\
\frac{Wp}{h} = \frac{0.1}{0.46} = 0.10 \\
\frac{S_1}{S_2} = 0.55 \quad \text{(from Fig.6)}
\]

\[
S_2 = 1.01 \times 0.55 = 0.556m
\]

**Fig. 3 - WCC Subsidence and Strain Curves over LII to IV**

This relationship is shown in Fig.4 which also includes data from some other mines on the South Coast as reported by Holle (1985).

**Fig. 4 - Effect of Chain Pillars in Reducing Subsidence**

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(ii) Calculate $S_{\text{max}_1}$

\[ W = 237\ m + 49\ m + 237\ m = 523\ m \]
\[ \frac{W}{h} = \frac{523}{465} = 1.15 \]
\[ S_{\text{max}_1} = \frac{L}{m} = 0.60 \text{ (from Fig.5)} \]
\[ S_{\text{max}_1} = 2.8 \times 0.60 = 1.680\ m \]

(iii) Calculate $S_{\text{max}_2}$

\[ \frac{W_p}{h} = 0.10 \]
\[ \therefore \frac{S_1}{S_2} = 0.55 \text{ (from Fig.6)} \]
\[ \therefore S_{\text{max}_2} = 1.680 \times 0.55 \]
\[ = 0.924\ m \]

To deduce the maximum subsidence for LWIII, a graph is drawn for LWI, II & IV with LWII being an interpolation.

Final maximum subsidence occurs when the theoretical width, as calculated, reaches the normal supercritical width (1.4h).

The value of final maximum subsidence can be determined by using the maximum value from Fig.5 and the reduction factor from Fig.6.

\[ S_{\text{max}_2} = m \times \frac{S_{\text{max}}}{m} \times \frac{S_1}{S_2} \]
\[ = 2.8 \times 0.63 \times 0.55 \]
\[ = 0.970\ m \]

Table 3 compares the results of actual predicted subsidence due to longwalls I and IV.

---

Fig. 5 - Subsidence for Varying $W/h$ Ratios

Fig. 6 - Reduction to Calculated Maximum Subsidence Due to Pillar Zones

LONGWALL I TO IV

Calculation of a theoretical width of LWI + LWII is made based on the predicted subsidence for LWI + LWII. This theoretical width of extraction is deduced from Fig.5, and will simulate a complete goaf with no intervening chain pillar.

(i) Calculate Theoretical Width

\[ S = \frac{0.556}{m} \times \frac{W}{h} = 0.51 \text{ (from Fig.5)} \]
\[ \therefore W = 465 \times 0.51 \]
\[ = 237\ m \]

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METHOD OF PREDICTION OF SUBSIDENCEprofiles

The use of empirical methods as described above and commonly used in Australia (Frankham and Hella, 1984 and Kapp, 1985) and the United Kingdom, predict maximum subsidence values fairly reliably. The subsidence profiles however, are more difficult to calculate.

A number of theoretical methods have been developed in Europe. Kratsch (1983) and others (Praca Zbiorowa, 1980) gave excellent details of these methods. For the purpose of this exercise, three methods of prediction of subsidence profiles and strains have been tried. These are:

1. Salustowicz Method
2. SMC Method
3. Superposition Method

The results obtained using these methods are discussed below.

SALUSTOWICZ'S PROFILE FUNCTIONS

Salustowicz's profile functions are based on the theory of bending of beams on an elastic foundation. Subsidence is derived by solving a flexure equation. Other components of subsidence are derived from the subsidence function (Borcki and Chudek, 1972 and others; Praca Zbiorowa, 1980). The results of the theory developed by Salustowicz are briefly given below.

Salustowicz's method was selected because this is the only method that predicts negative subsidence (lifting) at the edges of the excavations. Lifting at the edges of the longwalls, particularly in the Brennan's Creek Ban area, was observed at West Cliff Colliery. The various parameters of the subsidence trough, according to the Salustowicz theory, are discussed below.

Subsidence Function

Subsides above the goaf and seam are given separately as follows: (Fig. 7)

Above the goaf, i.e. \( x \geq 0 \)

\[
S = S_{\text{max}} - \frac{a^2}{8^2 + b^2} S_{\text{max}} e^{-a x} \left[ \frac{8-a}{b+a} \sin ax + \cos ax \right]
\]  

(1a)

Above the seam, i.e. \( x < 0 \)

\[
S = \frac{a^2}{8^2 + b^2} S_{\text{max}} e^{bx} \left[ \frac{8-a}{b+a} \sin bx + \cos bx \right]
\]  

(1b)

where \( a = \frac{1}{4} \sqrt{\frac{k}{\rho E}} \); 
\( b = \frac{1}{4} \sqrt{\frac{C}{\rho E}} \);

\[
E' = \frac{E}{1-v^2}
\]

E' — Young's Modulus of the rock mass;

\( v \) — Poisson's ratio of the rock mass;

J — Movement of the inertia of the strata;

k — reaction of the floor;

C — stiffness of the coal seam.

It can be seen in Fig. 7 that the phenomenon of upheaval is predicted above the unextracted seam by using Eq. (1b).

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Fig. 7 - Prediction of Subsidence Profile by Salustowicz's Profile Functions

Curvature and Horizontal Strain

(i) Curvature

Curvature is the second derivative of subsidence, i.e.

\[ K = \frac{d^2S}{dx^2} \]

Deriving from Eqs. (1) and (2) we get:

\[ K_1 = \frac{2S}{h_a} \max \left\{ \frac{\text{arc tan} \frac{h}{h_a}}{h} \right\}^2 \times \frac{\text{tg} \theta}{h_0} e^{-ax} \]

\[ K_2 = \frac{2S}{h_a} \max \left\{ \frac{\text{arc tan} \frac{h}{h_a}}{h} \right\}^2 \times \frac{\text{tg} \theta_2}{h_0} e^{ax} \]

\[ \sin \alpha x = \frac{h}{h_0} \cos \alpha x \]

\[ x > 0 \]  \hspace{1cm} (2a)

\[ \sin \beta x + \frac{h}{h_0} \cos \beta x \]

\[ x < 0 \]

where

\[ \text{tg} \theta_1 = \frac{H}{x_1} \]

\[ \text{tg} \theta_2 = \frac{H}{x_2} \]

(ii) Horizontal Strain

Strain is proportional to curvature. Thus,

\[ \varepsilon = K_{ho} \]

where \( h_{ho} \) - the vertical distance from the ground surface to the neutral axis, assume \( h_{ho} = \frac{1}{2} H \).

**Simplified Functions**

Let \( a = \beta \), which results in

\[ k = c \] (generally \( c \neq k \))

and \( x_1 = x_2 \) or \( \theta_1 = \theta_2 \).

This assumption brings about the following simplified functions for subsidence and curvature.

(i) Subsidence Functions

\[ S = S_{max} \left( 1 - \frac{1}{2} e^{-ax} \cos \alpha x \right) \]

\[ x > 0 \]  \hspace{1cm} (1c)

\[ S = \frac{1}{2} S_{max} e^{ax} \cos \alpha x \]

\[ x < 0 \]  \hspace{1cm} (1d)

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(11) Curvature

\[ K_1 = -S \max . a^2 e^{-ax} \sin ax \]
\[ x > 0 \]  
(2.c)

\[ K_2 = -S \max . a^2 e^{ax} \sin ax \]
\[ x < 0 \]  
(2.d)

The following illustrates the influence of \( a \) on the calculation of subsidence and curvature.

The Points With Subsidence = \( S \max \), or Zero

Referring to Eq.(1.c):

\[ S = S \max \]

only when \( x = + \infty \)

or \( ax = n\pi + \frac{\pi}{2} \) \((n=1, 2, \ldots)\).

From Eq.(2.c), it is known that when \( x = + \infty \), \( K_1 = 0 \).

However, if \( ax = \frac{\pi}{2} \). (for the sake of convenience, take \( n = 1 \)),

\[ K_1 = -S \max . a^2 e^{-\frac{3\pi}{4}} = -0.21 a^2 S \max \]

Similarly, according to Eq.(1.d), when

\[ x = - \infty \], or \( ax = n\pi + \frac{\pi}{2} \) \((n=1, 2, \ldots)\)

\[ S = 0 \]

From Eq.(2.d),

when \( x = - \infty \), \( K_2 = 0 \)

\[ ax = \frac{\pi}{2} \]

\[ K_2 = S \max . a^2 e^{\frac{3\pi}{2}} \quad 0.21 a^2 S \max \]

Thus, it is concluded that, except at infinity, at the points with subsidence of \( S \max \), or zero, corresponding curvature or strain are not zero.

The Points of Maximum Subsidence

Based on Eq.(1.c)

\[ \frac{ds}{dx} = \frac{1}{2} S \max . ax \cos ax + \sin ax \]

Let \( \frac{ds}{dx} = 0 \) gives

\[ x = + \infty \]

or \( \cos ax + \sin ax = 0 \).

i.e. \( ax = n\pi + \frac{\pi}{2} \) \((n=1, 2, \ldots)\).

As discussed before when \( x = + \infty \),

\[ K_1 = 0 \]

If \( ax = \frac{3\pi}{4} \),

\[ S \quad (at \ ax = \frac{3\pi}{4}) = S \max . \]

\[ (1- \frac{1}{2} e^{\frac{3\pi}{4}} \cos \frac{3\pi}{4}) \]

\[ = 1.05 S \max . \]

\[ K_1 \quad (at \ ax = \frac{3\pi}{4}) \]

\[ a^2 e^{-\frac{3\pi}{4}} \sin \frac{3\pi}{4} \]

\[ = 0.07 a^2 S \max . \]

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"It is interesting that the maximum subsidence derived from the equation is 1.05 times that of the real maximum subsidence \( S_{\text{max}} \). Again, except at infinity, at the points of 1.05 \( S_{\text{max}} \), curvatures or strains would not be equal to zero. Therefore, Salustowicz's profile function may be only suitable for the prediction of subcritical subsidence.

Points of Zero Strain

From Eq. (2.2) to have \( F_1 = 0 \), there are three possibilities,

\[
x = 0 ; x + + = \text{or} \ ax = n(n=1,2,3)
\]

Take \( ax = 2 \) as an example,

\[
S = S_{\text{max}}(1 - \frac{1}{2} e^{-2 \cos n}) = 1.02 S_{\text{max}}
\]

If the equations are used in reverse and the maximum subsidence of 1.02 \( S_{\text{max}} \) is taken at point:

\[
x = \frac{r}{s}
\]

the strain at this point is zero, which indicates the critical subsidence. Hence, to predict critical subsidence, the maximum subsidence has to be 1.02 \( S_{\text{max}} \) at point:

\[
x = \frac{r}{s}
\]

and the strain is zero at this point.

MWB Method

The method is adopted for mine layouts typical of modern longwall mining where the face length is subcritical and the intervening development pillars are as small as possible. The pillars reduce surface subsidence considerably and the extent of the subsidence reduction depends on the pillar width.

Based on the sequence of extraction, the maximum subsidence after the mining of each longwall can be predicted. The methods of prediction are well known (NCB, 1975). The procedure described earlier is based upon the NCB method.

The subsidence profile for a given width to depth ratio is determined from an empirically derived graph in terms of \( S \) and the horizontal distances from the point above the longwall centre in terms of the mining depth.

**PREDICTION OF HORIZONTAL STRAINS USING MWB METHOD**

Strain is proportional to subsidence and inversely proportional to depth. The proportionality constants for tensile and compressive strains are empirically represented on a graph based on the ratio of width to depth. Thus, the maximum tensile or compressive strain, over an extracted area is proportional to \( S/h \), \( S \) being the maximum subsidence.

Like a subsidence profile, the horizontal strain profile for a given width to depth ratio is plotted based on an empirically derived graph (or a corresponding table) which gives the strains, in terms of the maximum strains and the horizontal distances from the point above the longwall.
METHOD OF SUPERPOSITION OF CRITICAL
SUBSIDENCE PROFILES

The method is based upon the principle of superposition (Brauner, 1973). An illustration is shown in Fig. 8 in which $S_1$ is a complete critical profile. In this case a semi-infinite extraction area is from $A_1$ toward $A_2$. Re-establishing the seam with its other edge at $A_2$ would create an inverse critical profile $S_2$. In terms of the principle of superposition, the difference $(S_1 - S_2)$ would be equivalent to the profile resulting from the extraction area between $A_2$ and $A_1$. Similarly, subsidence profiles for longwall layouts can be obtained.

![Fig. 8 - Superposition of two Critical Profiles](image)

SUBSIDENCE PROFILES PREDICTED
BY THREE METHODS

1. Salustowicz's Profile Function

Here, the assumption is made that the point of half maximum subsidence is located directly above the two edges of a longwall (for the first longwall the points of half subsidence are based on actual measurements).

\[ a \text{ in Eqs. (1.c) and (1.d) is determined according to} \]
\[ a = \frac{T}{U} \text{ when } S = S_{\text{max}} \]
\[ a = \frac{2T}{U} \text{ when } S = 1.02 S_{\text{max}} \]

where $U$ is the extraction width. The maximum subsidence above the centre of the extraction is $S_{\text{max}}$, which is known from the survey (Line B). These values of $a$ have been used in calculating subsidence profiles (Fig. 9).

![Fig. 9 - Subsidence Profiles Predicted by Salustowicz's Profile Functions](image)

2. NCB Empirical Method

The maximum subsidence values for longwalls are adopted from the field observations (along Line E). Subsidence profiles are then plotted according to the empirical table in the NCB Handbook (NCB, 1975) in which the angle of draw is assumed to be 35°. The profiles are shown in Fig. 10.

![Fig. 10 - Subsidence Profiles Predicted by N.C.B. Empirical Method](image)
3. Superposition Method

For this method, the critical subsidence profile for a semi-infinite extraction area is based on the subsidence profile predicted for the five longwalls by both Salustowicz's Profile Function and NCB Empirical Method. The profiles predicted by the Superposition Method are shown in Fig.11 and Fig.12.

Comparing Fig.10 with Fig.9, the profiles predicted by NCB Empirical Method fit the measured profiles rather well. Comparing Fig.9 with Fig.3, it is clear that the profiles based on Salustowicz's Profile Function do not coincide with the measured profiles. However, the uplifting of the ground surface above the unextracted area is predicted by this method. This uplift was not predicted by any of the other methods considered.

As shown in Fig.10 and Fig.12, the profiles predicted by superposition in which the critical profile is based on the NCB Empirical Method, have shapes similar to those directly predicted by the NCB Empirical Method, but show a greater zone of maximum subsidence. The profiles predicted by superposition in which the critical profile is based on Salustowicz's Profile Function, coincide well with those directly predicted by Salustowicz's Profile Function (see Fig.9 and Fig.11).

PREDICTION OF HORIZONTAL STRAIN BY SALUSTOWICZ'S PROFILE FUNCTION AND NCB EMPIRICAL METHOD

As in the prediction of subsidence profiles, horizontal strain profiles for the successive longwalls are predicted by both methods (see Fig.13).

The profiles from the subsidence surveys, along lines E and F, are shown in Fig.3. Upheaval of the ground surface appeared above the unextracted area.

Fig.13 Predicted Horizontal Strain Profiles
STRATA MOVEMENT ABOVE THE LONGWALLS

Measurements showed that the maximum subsidence after the extraction of LWI was very small (95mm). After the completion of LWII, the maximum subsidence increased to 141mm, subsequently increasing gradually.

Two methods were considered for the calculation of strata behaviour in order to account for the very small amount of subsidence after the extraction of LWI.

The first attempt to calculate strata movement was by the elastic bending of a plate. The overburden strata of the longwalls were treated as a rectangular plate with all edges fixed and with uniform loading. However, the result did not seem satisfactory.

Secondly, a solution to the strata movement was tried by using the theory of an elastic beam with fixed ends and uniform loading, as follows:

\[ S = \frac{q}{24EI} \left( l^4 / 2 - 2l^3 / 3 + x^4 \right) \]

where

- \( S \) — deflection, i.e. subsidence
- \( q \) — uniform loading, \( q = \gamma Wh \)
- \( L \) — span of a beam, i.e. the length of a longwall block;
- \( E \) — Young's Modulus of the beam, i.e. rock mass (0.6-5.9 GPa)
  (Lama and Vutukuri, 1978);
- \( I \) — moment of inertia of the beam:
  \[ I = \frac{1}{12} Wh^3 \]

\( W \) — the width of the beam, i.e. the width of the longwall;

\( h \) — Depth of cover

\( \gamma \) — the density of the rock mass, 2.5t/m\(^3\).

When \( x = \frac{L}{2} \), \( S \) reaches a maximum value

\[ S = \frac{W}{32Eh^3} \]

Eq.(6)

In these calculations, \( L \) is based on the relationship \( L = 1.3H = 605m \), i.e. critical length for subsidence to fully develop in the longitudinal direction. The results of the calculations are given in Fig. 14. Young's Modulus \( E \) of 5095MPa was calculated based on \( S = 0.095m \) from LWI, and \( h = H = 465m \). The intact depth \( H \) is then calculated from subsidence.

Thus the movement of the overlying strata can be explained in the following way:

1. After LWI, but before LWII was extracted, the strata underwent purely elastic deformation. And gave only a small amount of surface subsidence.
2. During the extraction of LWII, breaking of immediate roof and main roof took place and a pressure arch was formed. The thickness of the intact rock mass was reduced. As a result surface movement increases significantly.
3. As mining progress from LWIII to LWIV, breaking and cracking continues both sideways and upwards. After the critical extraction width is reached, the breaking does not extend up any further.

CONCLUSION

1. The method of calculating maximum subsidence by introducing a theoretical width of goaf proved effective, but it relies on the accuracy of the graph in Fig. 6.
2. Subsidence profiles predicted using the NCB Empirical Method showed reasonable agreement with measured profiles, after maximum subsidence of the profiles was determined.
3. Uplifting of the ground at the extremity of the subsidence zone is predicted using Salustowicz’s Profile Function.
4. Production of subcritical profiles by superposition is effective only if the critical profile has been drawn. However, it has limited application in longwall mine layouts, as described in this paper earlier.

5. The theoretical height of the beam, as calculated, reduced dramatically as LWI was extracted. This demonstrated that the beam was either still intact, or only just failed after the extraction of LWI.

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