AN EVALUATION OF FRACTURE TOUGHNESS OF COAL MEASURES ROCKS

By
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ABSTRACT

This research work is concerned with an investigation of the fracture toughness of sandstone, siltstone, and coal. Tensile failure modes have been examined with three different specimen geometries, i.e., rectangular bar, round bar (in three point bending), and diametral compression discs. The Stress intensity approach, and J-integral methods were used to evaluate fracture toughness. Results from both approaches are discussed. True crack length was determined after pre-cracking, using the compliance calibration technique. Fracture toughness results of sandstone are independent of crack length and the size of the specimen. Fracture toughness may hence be regarded as a material property for sandstone. In contrast, fracture toughness values for siltstone and coal are dependent on crack length.

INTRODUCTION

Fracture toughness is a fundamental property of rock which governs the growth of fractures, within the rock under static or dynamic stresses. The role of fracture mechanics in the design of rock structures, such as underground openings of various types, surface mine slopes, and rock fragmentation, is vitally important. However, it has become customary to use the engineering properties approach in design of stable rock structures due to the complexities of rock structures, and lack of understanding of the fundamentals of failure mechanisms (Hardy 1973).

In mining engineering fracture mechanics may be used to calculate the formation of fracture zones around mine openings, thus estimating support requirements and formulating guide lines for the selection of mine roadway support systems. The main problem in effectively predicting support requirements is the evaluation of the physical extent and geometry of the fracture zones around mine roadways. If an accurate extent of the fracture zone could be predicted, then from a knowledge of the strata properties of the loosened ground, support loading could be determined.

The purpose of this research program is to determine the fracture toughness parameters of coal measures rocks in the United Kingdom.

EXPERIMENTAL PROGRAMME

The Stress intensity approach and J-integral technique have been used to evaluate the fracture toughness of coal measures rock, such as, sandstone, siltstone, and coal. In the stress intensity approach fracture toughness is measured in terms of stress intensity factor, K, which is a measure of stress intensity at a crack tip. K, essentially describes the entire stress field at a crack tip in a linear elastic material. When a sufficient level of stress intensity is applied to a material the crack will extend. This critical value of K is referred to as fracture toughness, Kc, when linear elastic conditions prevail.

The Stress intensity approach to evaluate fracture toughness parameters, is limited to linear elastic fracture mechanics, but on the other hand the J-integral concept provides the facility to solve fracture problems ranging from linear elastic to elasto-plastic types, (Rice, 1968, and Bagley, and Landes, 1971).

In this study of fracture mechanics, three different specimen geometries, i.e., (i) single edge cracked bar in three point bending (SEB), (ii) single edge cracked round bar in three point bending (SECB), (iii) diametral compression discs (DCD), were used to investigate the experimental evidence of inherent material properties. All the experiments were designed to monitor the fracture toughness of coal measures rock in tensile failure mode. A purpose built specimen
holding assembly (Figure 1) was used for three-point bending tests.

The main features of material resistance to failure are associated with the crackgrowth resistance curves (Figs. 2, 3, 4, 5). From these curves critical values of fracture toughness can be determined.

The objective of the experimental programme was to select a suitable testing method, which was reliable, economical, and pertinent to the facilities available in the department. In view of these factors, three testing methods previously mentioned were selected for the experimental work to determine the fracture toughness of 3 types of rock. The J-integral technique was used to validate the results of the fracture toughness test on sandstone.

In order to carry out the test of three different specimen geometries, Howden-3000 servo hydraulic testing machine was used. A load cell of 100KN capacity was used to monitor the load. This load cell was precisely calibrated within the range 0-80KN. A special type of platen was designed, through which load was applied on the three-point bending specimen, to eliminate the twisting effect caused by rectangular shape specimens.

**MATERIAL AND SPECIMEN PREPARATION**

Sandstone specimens were ordered as rectangular bars of four different sizes, having cross-sectional area of 25 x 25 mm², 35 x 35 mm², 45 x 45 mm², and 55 x 55 mm², and with lengths greater than 4 times the width. These rectangular bars were then surfaced on a surface grinder. A ten micron diamond cutting wheel was used to

![Diagram of specimen holding assembly for three-point bending test](image)

**Fig. 1** Specimen holding assembly for three-point bending test

![Graph showing notch length vs fracture toughness](image)

**Fig. 2** Notch length vs fracture toughness

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**Sandstone**

\[ k_1 = 0.586 - 0.182(\alpha/W) \]

**Fig. 3** Dimensionless Crack Length vs Fracture Toughness

**Siltstone**

\[ k_1 = -0.0236 + 6.143E-03(\alpha) \]

**Fig. 4** Notch Length vs Fracture Toughness

**Coal**

\[ k_1 = -0.135 - 9.56E-03(\beta_0) \]

**Fig. 5** Notch Length vs Fracture Toughness

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prepare straight notches in the specimen. In order to monitor the crack growth during testing, it was necessary to attach a linear variable differential transducer (L.V.D.T) of 0.25 mm range to the specimen, across the notch. For this reason, transducer holders, prepared in the workshop, were then glued on either side of the notch, to accommodate the transducer, as shown in Figure 6.

Ten cylindrical specimens were obtained from slatestone samples, brought from the Ryefield open pit coal mine, with diameters of 50 mm, and lengths of more than 3.33 times the diameter of the core. Samples were cored in a direction perpendicular to the bedding plane. Notches of different sizes were made in the centre of all core specimens with the help of a diamond cutting wheel. Transducer holders were glued to each specimen to mount the transducer across the notch.

A coal sample was brought from the Cadleyhill Colliery and casted in concrete, so that it could be cured properly. 50 mm cores were obtained from the coal, and 15 discs of 17 mm thickness were cut from these cores and surfaced. Notches were made through the diameter with a diamond cutting wheel. Seven specimens were broken during notch preparation.

**TESTING OF SPECIMEN IN THREE POINT BENDING**

The testing set up for three point bending tests is shown in Figure 7. In the case of a single edge rectangular bar, the span was adjusted to 4 times the thickness of the specimen. While in the case of a round bar in bending test, the distance between two supporting rollers was kept to 3.33 times the diameter. The machine was set in the strain control mode, to apply a crack growth of 2 μm/s. Each specimen was loaded and unloaded a few times at different strain levels, to initiate a sharp crack in the notch, and finally the specimen was loaded until failure. Test readings were recorded automatically at every 10 seconds, and data stored by the computer on a floppy disc, with a print out being obtained of the readings at the end of the test.

The basic requirement for the determination of fracture toughness, in three point bending tests are as follows—

(a) pre-cracking of the specimen

(b) obtain load Vs crack opening displacement curve
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(c) the development of compliance calibration graph

(d) determination of the true crack length after pre-cracking with the help of the compliance calibration graph

(e) select critical load (in the case of SEBMS specimens)

(f) select failure load (in the case of SEBMS specimens)

(g) calculate results.

PRESENTATION OF RESULTS

Calculation of \( k_{j5} \) values for sandstone (JPF)

Load Vs crack opening displacement graphs of each JPF test were plotted as shown in Figure 8. In this particular test the critical load \( P_{c5} \) is determined from the sixth cycle. It must be noted that at this stage the notch length changed during the previous five loading and unloading cycles. So while considering the sixth cycle for determination of critical load, \( P_{c5} \), it is necessary to take into account the increment of initial notch length. It may be represented as by the following equation:

\[
a = a_0 + a_i
\]

where

- \( a \) = new crack length after pre-cracking (mm)
- \( a_0 \) = initial notch length (mm)
- \( a_i \) = increment in initial notch length (mm)

The compliance calibration technique was used to find out the increment in the initial notch length, and has been a successful tool for finding out the increment in initial notch length, (Schmidt, 1976; 1977, and Schmidt and Hiddle, 1977). The compliance calibration graph developed is shown in Figure 9. A computer programme was written to find out the best fit regression curve showing the following empirical relationship:

\[
\lambda_{c5} = 7.7 + 16072(a_o/w)^{5.5} - 16452.2(a_o/w)^{5.7}
\]

where

- \( \lambda \) = compliance (mm/N)
- \( c_{5} \) = dimensionless crack length
- \( a_o \) = initial notch length (mm)
- \( w \) = width of the specimen (mm)

Critical load \( P_{c5} \) is then determined from load Vs crack opening displacement graphs, as shown in Figure 10, by using the 5% secant method. Fracture toughness values are calculated from the following relationship:

\[
K_{ic} = P_{c5} \times \frac{S \times 3.8^{1/2} \times W^{2}}{(2.9 - 4.6(a_o/w) + 21.8(a_o/w)^2 - 37.6(a_o/w)^3 + 38.8(a_o/w)^4)}
\]

where

- \( K_{ic} \) = fracture toughness in opening mode
- \( P_{c5} \) = critical load (KN)
- \( S \) = span length (mm)
- \( W \) = width of the specimen (mm)
- \( a_o \) = notch length after pre-crack (mm)
- \( B \) = thickness of the specimen (mm)

Fig. 8 CRACK OPENING DISPLACEMENT VS LOAD GRAPH SHOWING SIX CYCLES IN THREE POINT BENDING TEST

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Calculated values of fracture toughness for sandstone specimens, $K_c$, are plotted against crack length, $a$, and dimensionless crack length, $a/W$, as previously shown in Figures 2 and 1. The average values of fracture toughness for four different sizes of SS38 sandstone specimens are plotted against the average width size in Figure 11.

**CALCULATION OF J VALUES FOR SANDSTONE IN 3PM**

According to this method, work done by the machine on the specimen is calculated during loading to the selected value of the load line displacement, and by measuring the area under load displacement graph. Work done was calculated at different values of load line displacement, $V$, of 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.11 (mm), (Figure 12).

The results are plotted against notch length, $a$, as shown in Figure 12. These curves show the best fit relationships, through the data points. Best fit curve formulae obtained for these curves are as given as follows:

- For $V=0.03$
  - $W = 9.2659 - 0.028(a)^{-0.1} + 5.65513(a)^{10}$(4)
- For $V=0.04$
  - $W = 0.019 + 0.27(a)^{-0.1} + 5.59812(a)^{-10}$(5)
- For $V=0.05$
  - $W = 0.035 + 0.048(a)^{-0.1} + 14.194(a)^{-7.6}$(6)
- For $V=0.06$
  - $W = 0.054 + 0.075(a)^{-0.1} + 61.1(a)^{-5.5}$(7)
- For $V=0.07$
  - $W = 3.4738 - 0.0826(a)^{-0.1} + 61.1(a)^{-4}$(8)

**Fig. 10 CRACK OPENING DISPLACEMENT VS LOAD GRAPH FOR THE DETERMINATION OF CRITICAL LOAD**

**Fig. 9 COMPLIANCE CALIBRATION GRAPH FOR THE DETERMINATION OF CRACK LENGTH AFTER PRE-CRACKING**

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Fig. 11 AVERAGE WIDTH VS AVERAGE FRACTURE TOUGHNESS

For $V = 0.11$

$W_{av} = 2.80 \times 10^7 + 0.04(a)^0.7 + 44.6 \times 10^9(a)^{-0.4}$ (11)

With the help of these eight equations, the $J$-integral values can be calculated for each specimen by using the following relationship (Landes and Bagley, 1971, b):

$J = \frac{1}{B} \left( \frac{W_{av}}{d} \right)$ (12)

The results are plotted against load line displacement ($L/L_{D}$) for different notch lengths (Figures 13). According to the method for $J_{f}$ values, the results are determined from the figure 13 for each specimen at the load line displacement values ($V$) which correspond to the critical load in the load-displacement graph. The values of $J$-integral are plotted against crack length, $a$, in Figure 14.

RESULTS FOR SANDSTONE

Siltstone specimens were tested under single edge cracked round bar in three point bending (SRCTB) test conditions for the determination of fracture toughness. During the test, three of the specimens were broken along the grains of mica within the specimen. Therefore, these three results were not considered in the analysis. To calculate the fracture toughness values, $K_{f}$, the following relationship was used (Swan, 1980):

$K_{f} = \frac{3V_{m} \psi^{2}(a)^{0.5}}{B} \frac{1}{a^{3}}$ (13)

where

$K_{f} =$ fracture toughness (MN/m$^{3/2}$)
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Y = dimensionless stress intensity factor
P = failure load (KN)
a = crack length at failure (mm)
S = span length (mm)
D = diameter of the specimen (mm)

The formula for dimensionless stress intensity factor (Y) is given as follows:

\[ Y = 1.25(19.666(a/D)^{4.5} - 0.5)(1 - a/D)^{0.25} \]  \hspace{1cm} (14)

The crack length of the specimen at failure (a) is obtained by using the compliance calibration graph for siltstone as shown in Figure 15. Fracture toughness values are plotted against crack length (a) in Figure 4.

Fig.13 Load line displacement vs J-integral graph showing the best fit curves for various notch lengths.

Fig.14 NOTCH LENGTH VS J-INTTEGRAL GRAPH SHOWING CRITICAL J-INTTEGRAL VALUES

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Eight specimens were tested with different notch lengths. The results of fracture toughness, $K_I$, are plotted against notch length, $a$, as previously shown in Figure 5.

**DISCUSSION**

Linear regression was completed for fracture toughness values, $K_I$, of sandstone plotted against notch length, $a$, (Figure 2). This clearly shows that fracture toughness does not change with respect to notch length, $a$, and the best fit curve can be treated as a material property curve. However, the best fit linear regression curve for fracture toughness results of sandstone plotted against dimensionless crack length, $a/W$, (Figure 3) gives somewhat downward trend with respect to the dimensionless crack length, $a/W$.

The average values of fracture toughness, $K_I$, for four different sizes of sandstone specimens (Sample 1) were calculated, and are plotted against the average width of the specimens of four different sizes (Figure 11). The best fit linear regression line shows that fracture toughness does not change with respect to the size of the specimen. Hence it can be concluded that the fracture toughness of sandstone is independent of crack length and specimen size, and can be treated as a material property curve for sandstone.

Critical $J$-integral values of single edge cracked bar in three point bending specimens of sandstone are plotted against notch length, $a$, (Figure 14). The best fit curve for the data shows that the $J$-integral values first decreases drastically and then slowly as crack length, $a$, increases.

$J$ values can be converted to fracture toughness parameters and vice versa, using the following equation:

$$J = \frac{(1 - \nu^2)}{E} \frac{P_c^2}{a}$$

Young's modulus, $E$, and poisson's ratio, $\nu$, for sandstone were obtained in laboratory tests as 11.01 GPa and 0.11 respectively. Table 1 shows critical $J$-integral values, fracture toughness, $K_I$, values, calculated $J$ values from equation (16), and the difference of the square of the difference between critical $J$-values and calculated $J$-values. It is apparent from the table that actual $J$ values are much higher than calculated $J$ values for small notches. The discrepancy between actual $J$ values and calculated $J$ values decreases as crack length increases. The square of the difference between critical $J$ values and calculated $J$ values are plotted against notch length, $a$, on a log scale (Figure 16). Linear regression analysis was carried out for data and the best fit curve is shown in Figure 16. This curve cuts the $y$-axis at $a=14.16$ mm, at this value of notch length, critical $J$-integral value and calculated $J$ values are consistent. Critical $J$-integral value can be determined from Figure 2 at $a=14.16$ mm as $J_c=24.04$ J/m$^2$. At $a=14.16$ mm $K_I$ value can be obtained from Figure 14 for $K_I=40.482$ MN/m$^2$. Substitution of this value into the equation, gives the result of $J_c=17.64$ J/m$^2$, this calculated value of $J_c$ is compatible with the critical $J$-value of $J_c=24.04$ J/m$^2$ determined from Figure 14. Hence, it may be concluded that both, the stress intensity approach, and the $J$-integral technique give compatible results for sandstone.
Fracture toughness results of siltstone are plotted against notch length, a, (figure 4). The best fit curve shows that the fracture toughness increases as crack length increases.

Kc values for coal are plotted against notch length, and the best fit curve shows a slightly decreasing trend of fracture toughness as crack length grows (figure 5).

### Table 1

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<th>Spec. No</th>
<th>Notch crit.</th>
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<th>Jc (J/m²)</th>
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### Conclusions

From the foregoing discussion the following conclusions can be made:

- Fracture toughness of sandstone is independent of notch length and specimen size. The best fit regression curve can be treated as material resistance curve.

- The Stress intensity approach, and J-integral technique give compatible results for sandstone.

- Fracture toughness of siltstone and coal are dependent on notch length.

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### References


