A STUDY ON STRESS DISTRIBUTION AND REASONABLE SIZE OF COAL PILLAR IN A COAL FACE

By

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ABSTRACT

Mechanical models are established in accordance with the degree of mining on both sides of the coal pillar, and formulae were derived which calculated the superposition stress distribution and the width of the plastic region. The stability of the coal pillar was analysed and a reasonable size of coal pillar determined. In accordance with the elastic region rate. Further practical examples for the stability of the coal pillar are calculated and simulated.

INTRODUCTION

During production in the coal mine, many underground structures such as the roadway, and the overburden strata of the working face need the protection of the coal pillar. The stability of the coal pillar depends on whether each structure is normally in use and attains the expected goal. It is important that a reasonable size of the coal pillar be correctly determined to ensure its stability.

MECHANICAL MODEL OF THE ROOF STRATA

MOVEMENT AND STRESS IN COAL WALL

1. When the width of the working face 1>H/3 (H is the mining depth), some groups of stable strata.

The differential equations of bending deflection of the roof strata are:

\[ \frac{d^4Z}{dx^4} = \frac{P_x - K(Z - Z_0)}{EJ} \quad x > 0 \]  

\[ \frac{d^4Z}{dx^4} = \frac{P_x + CZ}{EJ} \quad > 0 \]

With consideration for the displacement boundary conditions, and the continuous conditions, equations (1) and (2) are combined to calculate the bending deflection.

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equations of the roof strata on the waste pack and the coal seam.

\[
Z = \frac{P_x}{K} + Z_0 + e^{\frac{P_x}{K}}(C_x \sin \theta x + C_y \cos \theta x), \quad x < 0
\]

\[
Z = \frac{P_x}{K} + e^{\frac{P_x}{K}}(B_x \sin \theta x + B_y \cos \theta x), \quad x > 0
\]

Where: \(\alpha = \frac{\sqrt{2}E_x}{\sqrt{4EJ}}, \text{ m}^{-1}\); \(\beta = \frac{\sqrt{2}E_x}{4EJ}, \text{ m}^{-1}\);

\[
Z_0 = \frac{P_x}{C} + \frac{\alpha^2 P_x}{\beta K}, \text{ m};
\]

\[
C_1 = \frac{\beta \alpha P_x}{\beta + \alpha}, \text{ m};
\]

\[
B_1 = \frac{\alpha^2 \beta}{\beta + \alpha}, \text{ m};
\]

\[
C = \frac{E_x}{(1 - \mu)K}, \text{ N/m};
\]

\[
K = \frac{E_x}{(1 - \mu_0)P}, \text{ N/m};
\]

\[
p = \frac{1}{1 - Kp};
\]

\(Kp\): The waste bulkiness factor.

\(E_x\): The elastic modulus of coal seam, N/m².

\(E_o\): The elastic modulus of waste pack, N/m².

When equation (4) is timed by factor C, the stress distribution in the coal seam will be calculated by:

\[
\sigma_x = \frac{P_x}{C} + e^{\frac{P_x}{K}}\left(\frac{\beta \alpha P_x}{\beta + \alpha} \sin \theta x + \cos \theta x\right)
\]

(5)

If the direction towards the coal seam is taken to be positive, then equation (5) may be rewritten as:

\[
\sigma_x = \frac{P_x}{C} + e^{\frac{P_x}{K}}\left(\frac{\beta \alpha P_x}{\beta + \alpha} \sin \theta x - \cos \theta x\right)
\]

(6)

2. When the width of the working face is smaller, \(L < H/3\), and there is not sufficient movement of stable rock owing to mining, the mechanical model shown in Fig. 2 could be applied.

**Fig. 2 Model of the Roof Strata Bending**

The co-ordinate system is set up as shown in Fig. 2. The differential equations of the bending deformation of the rock beam are given by:

\[
EJ \frac{d^2Z}{dx^2} = P_x - C Z(x), \quad x > 1
\]

(7)

\[
EJ \frac{d^2Z}{dx^4} = P_x, \quad 0 < x < 1
\]

(8)

Based on the displacement boundary conditions and the continuous conditions, equations (7) and (8) are combined to work out the bending and subsiding curve equations of the rock beam.

\[
Z(x) = \frac{P_x}{C} + e^{\frac{P_x}{K}}(A \sin \beta x + B \cos \beta x) \quad \text{for } x > 1
\]

(9)

\[
Z(x) = \frac{P_x}{24EI} e^{\frac{P_x}{K}} x + \frac{C + D}{2EI}, \quad 0 < x < 1
\]

(10)

Where:

\[
A = \frac{P_x^3}{3C} \beta : \beta + 3 \beta^2 + 3 \beta + 3, \text{ N};
\]

\[
B = \frac{P_x^3}{3C} \beta : \beta + 2 \beta^2 + 6 \beta + 3, \text{ N};
\]

\[
C = \frac{P_x^3}{6(1 + \mu)C} \beta : \beta + \mu^2 \beta + 3, \text{ N};
\]

\[
p = \frac{P_x}{24EI} \beta : \beta + \frac{C}{2EI} \beta^2, \text{ N/m}².
\]
With equation (9) timed by C, the stress distribution inside the coal wall is given by:

\[ \sigma_1 = P + P_x \frac{1 - \beta}{(3 + 1 + \beta)} \sin \beta x + (2 + 6 \beta + 3 \cos \beta x - (1 + \beta))e^{\beta x} \quad x > 0 \quad (11) \]

If the co-ordinate zero point is moved to the boundary of the coal wall, then equation (11) can be rewritten as:

\[ \sigma_2 = P + P_x \frac{1 - \beta}{3(1 + \beta)} \sin \beta x + (2 + 6 \beta + 3 \cos \beta x) e^{\beta x} \quad x > 0 \quad (12) \]

**THE WIDTH OF PLASTIC REGION IN COAL WALL**

When mining stresses in the coal wall, \( \sigma_1 \) are equal to the ultimate strength of coal \( \sigma_{um} \), the value of \( x \) towards the direction of the coal wall would be equal to the plastic region width \( R_p \). Furthermore, the weight of overburden stress is not considered to influence the stability of the coal pillar, thus the width of plastic region can be determined as follows:

When \( L < H/3 \), then:

\[ \sigma_{um} = P \sqrt{C_R \frac{P - \sigma}{C_R \frac{1 - \alpha}{\beta} \sin \beta x + \cos \beta x} e^{\beta x}} \quad (13) \]

When \( L > H/3 \), then:

\[ \sigma_{um} = P \frac{1 - \beta}{3(1 + \beta)} \sin \beta x + (2 + 6 \beta + 3 \cos \beta x) e^{\beta x} \quad (14) \]

Equations (13) and (14) can be used to calculate the plastic region of the coal pillar \( R_p \), and an alternative method used when only needing to know parameters \( C_R, \beta, \alpha \), and \( \sigma_{um} \).

The yielding strength of the coal pillar is based in Mises' yielding law in the plasticity theory which indicates that material is yielding under conditions of axial stress. The classification of coal hardness is shown in Table 1.

**Table 1: Actual compression strength of coal wall & classification of coal**

<table>
<thead>
<tr>
<th>Classification value</th>
<th>Soft coal</th>
<th>Medium coal</th>
<th>Hard coal</th>
<th>Extreme hard coal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial yielding compressive stress ( \sigma_1 ) (MPa)</td>
<td>5.58</td>
<td>15.68</td>
<td>25.48</td>
<td>35.28</td>
</tr>
<tr>
<td>Restrain index ( \delta )</td>
<td>1.68</td>
<td>1.3</td>
<td>1.1</td>
<td>/</td>
</tr>
<tr>
<td>( \sigma_{um} ) (MPa)</td>
<td>6.91</td>
<td>14.27</td>
<td>28.03</td>
<td>/</td>
</tr>
</tbody>
</table>

However, the stress condition in the coal pillar is actually between an axial and a tri-axial state, so the coal pillar bears restraint to some extent. Therefore, the restrained plastic index \( \delta \) of the coal rib should be considered, as shown in Table 1. Furthermore, the actual yielding strength at the site, is lower than the compressive strength of the coal sample in the laboratory, which, in general, is 0.7 times one. Thus, the actual strength index of the coal wall should be:

\[ \sigma_{um} = 0.75 \sigma_{um} \quad (15) \]

**SUPERPOSITION OF THE STRESSES IN THE COAL PILLAR AND DETERMINATION OF STABLE SIZE OF COAL PILLAR**

Three main situations are identified on both sides of the coal pillar:

1. Degree of mining on both sides of the coal pillar is higher, with mining width \( L > H/3 \).
2. Degree of mining in one side of the coal pillar is higher, \( L > H/3 \), and that in the other side of the coal pillar is lower, \( L < H/3 \).
3. Degree of mining on both sides of the coal pillar is lower, \( L < H/3 \).

When both sides of the coal pillar are affected by mining, the cover load is transferred on both sides of the coal pillar. This causes an increase in the width of plastic...
zones on both sides of the pillar. The extent of the plastic zones' increase is dependent upon the size of the pillar. The smaller the coal pillar, the wider the plastic zone will be. Therefore, to achieve coal pillar stability in the long term, it is necessary to ensure that the elastic region in the coal pillar occupies an appropriate proportion. Based on the relevant information, the elastic ratio in the stable coal pillar could be given by:

$$\frac{B - 2r}{p} = \begin{cases} \frac{P_{\text{inf}}}{b} \geq 0.65 \\ \frac{P_{\text{massed}}}{b} \geq 0.85 \\ \frac{P_{\text{hast}}}{b} \geq 0.90 \end{cases} \quad (16)$$

Where $R_p$ - the plastic region width, m;
$B$ - the width of coal pillar, m.

As the elastic ratio is considered, it is simplified to include simple stress in value instead of overlapping it with stress state.

Now consider the following three situations:

1. The degree of mining in both sides of the coal pillar is higher, $L > H/3$.

Distribution of the incremental stress in the coal pillar:

$$\Delta \sigma(x) = \frac{P_{\text{inf}}}{b} \sqrt{\frac{C}{K}} \left( \frac{\beta - \alpha}{\beta + \alpha} \sin \beta x + \cos \beta x \right) e^{-\frac{x}{b}}$$

$$+ \frac{P_{\text{massed}}}{b} \sqrt{\frac{C}{K}} \left( \frac{1 - \beta - \alpha}{\beta + \alpha} \sin \beta x + \cos \beta x \right) e^{-\frac{x}{b}} \quad (17)$$

Let equation (17) equal $\sigma_{\text{eq}}$ which is combined with equation (16). The alternate method could be used to calculate the plastic region width $R_p$ and the reasonable size of the coal pillar.

2. The degree of mining in one side is higher, $L > H/3$, while another side is lower, $L < H/3$.

Distribution of the incremental stress in the coal pillar is:

$$\Delta \sigma(x) = \frac{P_{\text{inf}}}{b} \sqrt{\frac{C}{K}} \left( \frac{\beta - \alpha}{\beta + \alpha} \sin \beta x + \cos \beta x \right) e^{-\frac{x}{b}} + \frac{P_{\text{hast}}}{b} \sqrt{\frac{C}{K}}$$

$$+ \frac{1}{2} \left( \frac{1 - \beta + \alpha}{\beta + \alpha} \sin \beta x + \cos \beta x \right) e^{-\frac{x}{b}} \quad (18)$$

Where $D = \frac{L_{\beta} B}{3(1 + L_{\beta} B)}$

$$+ (2L_{\beta} B^2 + 6L_{\beta} B + 3) \cos \beta x \quad (B-x) e^{-\frac{B-x}{b}}$$

$L_{\beta}$ - half width of face where the degree of mining is lower in one side of the coal pillar.

Let equation (18) equal $\sigma_{\text{eq}}$, with which the plastic region width can be calculated.

Where the degree of mining on one side is higher, the formula for the plastic region of the coal pillar is:

$$x = \frac{L_{\beta}}{\beta} \left( \sqrt{\frac{C}{K} \left( \frac{\beta - \alpha}{\beta + \alpha} \sin \beta x + \cos \beta x \right)} + \frac{1}{P_{\text{D}}} \right) \quad (19)$$

In another side, the formula for calculating the coal pillar plastic region is:

$$x = \frac{1}{\beta} \left( P_{\text{D}} \sqrt{\frac{C}{K} \left( \frac{1 - \beta - \alpha}{\beta + \alpha} \sin \beta x + \cos \beta x \right)} + x \right) \quad (20)$$

Where: $F = \frac{L_{\beta} B}{3(1 + L_{\beta} B)}$

$$+ (2L_{\beta} B^2 + 6L_{\beta} B + 3) \cos \beta x$$

Equations (19) and (20) are alternate formulae. After $x$ is chosen, $x_{1}(R_{p1})$ and $x_{2}(R_{p2})$ are calculated. $x_1$ and $x_2$ are substituted into equations (19) and (20) to work out $x_1$ and $x_2$, $B$ is calculated with equation (16) again and repeated until $(R_{p}-B_{m-1})$.

Finally, the value of $B$ attained is equivalent to the size of the coal pillar between the adjacent workings.

3. The degree of mining on both sides of the coal pillar is lower, $L < H/3$.

Distribution of the incremental stress in the coal pillar is given by:

$$\Delta \sigma(x) = \frac{P_{\text{inf}}}{b} \sqrt{\frac{C}{K}} \left( \frac{\beta - \alpha}{\beta + \alpha} \sin \beta x + \cos \beta x \right) e^{-\frac{x}{b}} + \frac{1}{2} \left( \frac{1 - \beta + \alpha}{\beta + \alpha} \sin \beta x + \cos \beta x \right) e^{-\frac{x}{b}} \quad (21)$$

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Let $\Delta \sigma(z) = \sigma_{th}$. The reasonable width of the coal pillar and its plastic region width can be calculated as above.

In the meantime, the ratio of width to height required is $B/m > 5$, which ensures that the stable condition of the pillars is satisfied.

It can be seen from the equations above, that the factors which affect the reasonable size of the coal pillar are:

1) physical and mechanical properties of coal and rock;
2) the degree of mining on both sides of the coal pillar;
3) the condition of the roof and floor strata in the goaf;
4) the elastic rate of coal pillar;
5) the ratio of width to height of the coal pillar.

The degree of mining on both sides of the coal pillar and the elastic rate are prevailing factors.

A PRACTICAL EXAMPLE

The conditions of coal mining in the Fourth Coal Mine, Pin Shing Shan Coal Mining Bureau are discussed.

There are three mineable coal seams under four living houses in Fourth Coal Mine; they are seam 15, seam 16-17 and seam 20. The average depth from the surface is 153m for seam 15. Seams 16-17 were mined using the longwall method and mining heights were 1.3m-1.5m and 3.5-3.8m respectively. Mining took place between 1964 and 1979. Four reinforced concrete buildings, with 4-5 floors, were constructed in the boundary of a subsidence basin caused by repeated mining. It is now planned to extract the coal face 2080 in seam 20. The coal face 2060 in the upper sub-level has been extracted by longwall mining. There was a sub-level coal pillar 26m wide between coal face 2060 and 2080.

The thickness of coal seam 20 is 1.7m, the dip angle 18°, the thickness of alluvial deposit 63m and the average buried depth 211m. The interval between seam 20 and seams 16-17 is 58m. The strata is mainly hard rock.

Of the schemes put forward for mining underbuildings in the Fourth Coal Mine, Scheme 1 was to extract 34m wide pillars on the tail entry of 2080 face and to extract upward coal pillar 6m long. To discuss the stability of the coal pillar, the formulae calculated above are used.

On the basis of information from the site, it is known that the axial compression strength of coal seam $\sigma_{th} = 16\text{MPa}$, belonging to the medium hardness, $E_0 = 1.47 \times 10^4 \text{MPa}$, $\mu = 0.3$, the bulking factor of the destroyed rocks $K_p = 1.6$, the compaction factor $K_p = 1.10$, its elastic modulus $E_0 = 0.588 \times 10^4 \text{MPa}$, the thickness of the main roof strata in No. 2060 face is 5m, $E = 3.66 \times 10^4 \text{MPa}$.

According to equation (15), the convergence index of the coal strength can be calculated as follows:

The elastic foundation factor of the coal pillar and the caved rock are:

$C = 9.302\text{MN/m}^3, \quad K = 277\text{MN/m}^3$

The degree of mining in No. 2060 face on one side of the coal pillar is different from No. 2080 face on the other side. The width of No. 2060 face is about 0m, which is more than $H/3$ ($H = 185m$), while No. 2080 face is about 60 metres wide, which is less than $H/3$ ($H = 198m$).

Therefore it is considered that the roof strata of the lower portion of No. 2060 face should sufficiently subside and the degree of mining be high and that the major roof strata in the lower portion of No. 2080 face should not fully sag, and the mining degree be low. Equations (19) and (20) determine the width of the plastic region due to mining in faces No. 2080 and 2060 on either side of the coal pillar, and the elastic ratio is checked with equation (16).

Given parameters are:

$\alpha(60) = 0.116\text{m}^{-1}, \quad \beta(50) = 0.281\text{m}^{-1},$

$\beta(80) = 0.0515\text{m}^{-1}, \quad V/C = 5.587, \quad \sigma_{th} = 14.56\text{MPa},$

$P_s = 4.95\text{MPa}, \quad M = 1.7m, \quad p = 185\%.$

Scheme I extracts 60 metres wide panel near the original tail entry and leaves a coal pillar of 20m. Scheme VI extracts upward of the original head entry 60 metres, given that seams nos. 15 and 16-17 have been fully extracted and No. 2060 face has also been extracted.
Table 2 Results of calculation

<table>
<thead>
<tr>
<th>scheme</th>
<th>working width (m)</th>
<th>width of coal pillar (m)</th>
<th>Rp - 2060 (m)</th>
<th>Rp - 2080 (m)</th>
<th>Elastic ratio (%)</th>
<th>Preload (%)</th>
<th>Stability</th>
<th>order</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>60</td>
<td>20</td>
<td>5.30</td>
<td>3.47</td>
<td>56.15</td>
<td>85</td>
<td>No</td>
<td>6</td>
</tr>
<tr>
<td>II</td>
<td>60</td>
<td>26</td>
<td>4.16</td>
<td>3.15</td>
<td>71.88</td>
<td>85</td>
<td>No</td>
<td>5</td>
</tr>
<tr>
<td>III</td>
<td>34</td>
<td>20</td>
<td>4.61</td>
<td>0.20</td>
<td>75.55</td>
<td>85</td>
<td>No</td>
<td>4</td>
</tr>
<tr>
<td>IV</td>
<td>34</td>
<td>26</td>
<td>3.83</td>
<td>0.08</td>
<td>84.56</td>
<td>85</td>
<td>Yes</td>
<td>2</td>
</tr>
<tr>
<td>V</td>
<td>30</td>
<td>20</td>
<td>4.26</td>
<td>-2.06*</td>
<td>78.70</td>
<td>85</td>
<td>No</td>
<td>3</td>
</tr>
<tr>
<td>VI</td>
<td>30</td>
<td>26</td>
<td>3.64</td>
<td>-2.12*</td>
<td>86.00</td>
<td>85</td>
<td>Yes</td>
<td>1</td>
</tr>
</tbody>
</table>

* Note minus presents coal wall doesn't enter the plastic state.

Table 2 shows the six types of calculation.

It can be seen from the calculations that the original scheme I, which extracts a 60m wide panel and leaves a coal pillar of 20m width, produces the worst results. In this case, the coal pillar could not remain stable in the long term; it gives rise to an instability phenomenon which causes faces Nos. 2060 and 2080 to collapse with each other, the roof strata to fully collapse and the surface buildings to be seriously affected. Thus, the calculations show that the original Scheme I should be replaced with scheme IV with respect to the coal pillar stability and the recovery rate of resource.

**SIMULATION TEST BY USE OF SIMILAR MATERIAL**

Two tests were conducted, one to research Scheme I and another to research Scheme VI. Models were designed as shown in Fig. 3.

![Fig. 3 Design of the plane model](image)

The surveying points on the observe and reverse sides of the model are as shown in Figs. 4 and 5.

![Fig. 4 Arrangement of surveying points on the observe side of the model](image)

![Fig. 5 Arrangement of surveying points on the reserve side of the model](image)

Firstly, extraction of seams nos. 15 and 16-17 was simulated. After the overburden movements ceased, the simulation of the No. 2060 face extraction in No. 20 coal seam was conducted.

In the simulated scheme I, the subsidence factor q was equal to 0.82. The caved zone is not obvious, but the
separated and fractured zones are comparatively obvious, and the height of greatest separation is 28 metres. The caved zone thickness is approximately 4-6 times the thickness of the coal seam, which is considered as 10 metres, so the separated zone is 38 metres thick. The whole caving zone is 30 metres thick. The rock interlayer between nos. 16-17 and no. 20 seams has not been fully destroyed due to an extracted coal face of 60m in length, which might be attributed to some form of mechanical balance as shown in Fig. 6.

It can be seen from the subsiding factor obtained, that the overburden movements are serious. This explains that the coal pillar width of 20 metres could not bear overburden pressure on both sides and gives rise to instability and subsidence of the overburden.

Fig. 6 Mining plane picture of Scheme I

In the scheme IV, the simulated mining subsidence is 1.28. When the JUAN face is extracted, the caved zone is 10 metres high, the vertical interval between the seam zone is 14.8 metres high, and the whole subsiding zone is 33.2m high. This means that some mechanical balance structures, which bear overburden pressure to stop the roof strata from moving, exist in the rock pillar between 16 - 17 and 20 seams. The smaller subsidence factor means that the coal pillar is more stable, and the whole subsides very little. Therefore, few movements due to mining are formed, as shown in Fig. 7.

The tests indicate that, with the same extraction width of 60 metres, in Scheme I the coal pillar easily loses its stability as its size is small. Faces nos. 2060 and 2080 connect through, which makes the whole subsidence more serious and the subsidence factor great. In Scheme VI, as the size of the coal pillar is great, its stability is good, and the whole subsidence is not serious, and the subsidence factor is small.

Test I also confirms that the calculations for Scheme I are correct.

Data obtained from the simulated tests for Schemes I and VI are shown in table 3.

Fig. 7 Mining plane picture of Scheme IV

From the parameters obtained from similar simulations, it can be seen that the parameters of Scheme IV are more ideal than Scheme I. The stability of the coal pillars in each scheme is different.

<table>
<thead>
<tr>
<th>Table 3 Results of the simulation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>$h$</td>
</tr>
<tr>
<td>$H_{1}$</td>
</tr>
<tr>
<td>$H_{2}$</td>
</tr>
<tr>
<td>$H_{3}$</td>
</tr>
<tr>
<td>$H_{4}$</td>
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<td>$H_{6}$</td>
</tr>
<tr>
<td>$H_{7}$</td>
</tr>
<tr>
<td>$H_{8}$</td>
</tr>
</tbody>
</table>

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CONCLUSIONS

Based on the above analyses, the main conclusions are:

1. The stress distribution in the coal pillar is related to the property of overburden strata, which includes hardness of the coal and floor rock, the degree of mining and the foundation factor, etc.

2. The stress distribution in the coal pillar is related to the property of coal itself and its geometric size.

3. The increased width of the plastic zones in both sides of the coal pillar are affected by the overlapped stress.

4. The long-term stability of the coal pillars is directly related to the elastic rate.

5. Using the appropriate formula contained in the paper, the reasonable size of the coal pillar could be determined, the plastic region width could be calculated, and the stability of the coal pillar analysed.

6. Simulations with similar material shows that surface movements are related to the size and stability of the coal pillar between adjacent seams and to the degree of mining on both sides of the coal pillar.

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