INFLUENCE OF THE SLOPING OF GROUND SURFACES ON MINE SUBSIDENCE

By

D. M. Shu1 and A. K. Bhattacharyya2

ABSTRACT

A two-dimensional finite element modelling technique is applied to predict the subsidence movements on a sloping ground surface above a completely mined panel. The modelling is carried out for the surface both directly and using a combination of an equivalent horizontal surface and a rays projection method and the result compared. The rays projection method provides the subsidence components on a sloping surface from the corresponding ones on an assumed equivalent horizontal surface through the point of mean elevation of the relevant part of the sloping surface. This method is used next to compare the subsidence, horizontal displacements and horizontal strains from the mining of a given panel at the surface for different inclinations. The results show that the magnitudes of the subsidence components are greater on the dip side of the slope from the point of the mean elevation, than the corresponding ones on the rise side and the asymmetry increases with the inclination of the surface. The findings are also compared with those of other research workers.

INTRODUCTION

The prediction of surface subsidence due to underground coal mining is often carried out by assuming a flat ground surface. In actuality however, the surface topography such as steep slopes, bill sides and valleys often exists in coal mining areas. Some in-situ measurements at such areas have indicated that surface topography appreciably affects the subsidence (Gentry and Abel, 1978; Asmoei and Jenan, 1982; Allgaier, 1982; Khair et al., 1987; Peng et al., 1987). Numerical modelling and analytical studies have also shown that surface topography has significant influence on the horizontal components of surface subsidence (Franks and Geddes, 1984; Holt and Mikula, 1984; Siriwardane and Amanat, 1984; Bowders and Lee, 1988, Whittaker and Reddish, 1989). Therefore, existing conventional methods of subsidence prediction for flat ground surfaces may not be appropriate for subsidence prediction for sloping ground surfaces.

A rays projection method and analytical equations have recently been developed by the authors of this paper (Shu, 1990; Shu and Bhattacharyya, 1992) to define the subsidence components on a sloping ground surface from the corresponding ones on an assumed equivalent horizontal surface through the point of the mean elevation of the sloping surface above an extracted panel. The subsidence components on the equivalent horizontal surface may be predicted by any existing method e.g. empirical method.

This paper analyses the effect of sloping ground surfaces on subsidence by using the rays projection method and a finite element numerical modelling technique together with data from field observations by other researchers. The patterns of subsidence movements for different ground slopes are examined.

DETERMINATION OF SUBSIDENCE ON A SLOPING GROUND SURFACE FROM THAT ON THE ASSUMED EQUIVALENT HORIZONTAL SURFACE USING THE RAYS PROJECTION METHOD

In the case of a sloping ground surface, an

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equivalent horizontal surface is first assumed at the position of the mean elevation of the sloping surface. The subsidence profile at the equivalent horizontal surface in a vertical transverse section can be predicted by any existing method of the mean elevation of the sloping surface. The subsidence profile at the equivalent horizontal surface in a vertical transverse section can be predicted by any existing method e.g., the National Coal Board's empirical method (National Coal Board, 1975).

Based on certain assumptions, it is then possible to project that profile on to the sloping ground surface using the rays projection method as illustrated in Fig. 1. The assumptions include those of linear subsidence limits through the strata overlying the extracted panel, movement of the undisturbed strata within the influence zone of the panel towards the worked-out void and the constancy of the volume subsided (or area, in a 2-D vertical section). Then, the subsidence components on the sloping ground surface associated with the extraction of a panel in a horizontal seam can be related to the corresponding subsidence components on the assumed equivalent horizontal surface as below:

Subsidence

\[ s(x) = (1-\mu)g(x) \]  

Tilt

\[ g(x) = (1-\mu)k(x) \]  

Curvature

\[ k(x) = (1-\mu)u(x) \]  

Horizontal displacement

\[ u(x) = (1-\mu)v(x) \]  

Horizontal strain

\[ e(x) = (1-\mu)\sigma(x) \]  

where

\[ x = \frac{x_c}{1-\mu} \]  

\[ \mu = \frac{\tan \beta}{H + \frac{W}{2 \cos \gamma}} \]  

\[ s(x), g(x), k(x), u(x), \sigma(x) \] are respectively the subsidence, tilt, curvature, horizontal displacement and horizontal strain on the sloping ground surface;

\[ s_c(x_0), g_c(x_0), k_c(x_0), u_c(x_0), \sigma_c(x_0) \] are respectively the subsidence, tilt, curvature, horizontal displacement and horizontal strain on the assumed equivalent horizontal surface;

W: extraction width;

H: mean extraction depth;

\[ \beta \]: angle of the inclination of the ground surface;

\[ \gamma \]: angle of draw;

\[ x \]: horizontal distance from the point on the sloping ground surface to the intersection point between the sloping surface and the equivalent horizontal surface and

\[ x_c \]: horizontal distance on the equivalent horizontal surface corresponding to \( x \) governed by the ray projection.

**MODELLING OF SUBSIDENCE ON A SLOPING GROUND SURFACE USING A NUMERICAL TECHNIQUE AND THE RAYS PROJECTION METHOD**

A two-dimensional, plane strain, finite element computer program called DEMON (Watson, 1988) is employed to simulate the subsidence due to the extraction of an assumed panel occurring on a sloping ground surface and the assumed equivalent horizontal surface.

Two meshes are generated. Mesh I, shown in Fig. 2, directly simulates a sloping ground surface at the chosen

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**Fig. 1** Projection of subsidence along the assumed equivalent horizontal surface on to the sloping ground surface by the rays projection method (Shu and Bhattacharya, 1992)

**Fig. 2** Mesh I—a FEM model to simulate subsidence on a sloping ground surface due to the extraction of a panel
angle of inclination of 18.4° (i.e. 1 in 3). The inclined surface is restricted to a finite length due to the capability of the program. The sloping part of the surface is directly above the extracted panel of width of 200 m and height of 3 m. The seam extracted is assumed to be flat. The extraction depth for the panel is between 200 m and 300 m due to the inclination of the ground surface, with the mean depth directly above the centre of the panel being 265 m. As in any other two-dimensional numerical modelling, the inclination and the extracted panel are assumed to extend to the infinity in the third dimension.

Mesh II, shown in Fig. 3, is modified from Mesh I and has the same total overburden weight as the latter. It is the model for the equivalent level ground condition with the extraction depth of 265 m. Extraction width, height and other conditions are the same as those in Mesh I.

![Mesh II model](image)

**Fig. 3** Mesh II—a FEM model to simulate subsidence on the equivalent horizontal surface due to the extraction of the panel

For the rock mass surrounding the mined panel in both sloping and level ground conditions, an Young's modulus of 10 GPa and Poisson's ratio of 0.25 and a pre-mining horizontal to vertical stress ratio of 1.0 are assumed.

The following steps are adopted in the modelling:

i) Directly modelling the subsidence on the sloping ground surface using Mesh I.

ii) Modelling the subsidence on the assumed equivalent horizontal surface using Mesh II and then, projecting it on to the sloping ground by the rays method using Eqs. (1), (6) and (7). The nominal angle of draw of 35° is assumed in the projection.

iii) For the purpose of comparison, the subsidence on the equivalent horizontal surface predicted in ii) is also directly projected on to the sloping ground surface ignoring any change of subsidence magnitudes, i.e.

\[ s(x) = s_0(x) \]

**ANALYSIS OF THE RESULTS**

The profiles of the different predictions of subsidence shown in Fig. 4 relate to the sloping ground surface. Profile A is the subsidence on the sloping surface directly predicted by program DEMON using Mesh I. Profiles B and C are respectively the subsidence projected on to the sloping surface according to Eqs. (1) and (8) from the one predicted for the equivalent horizontal surface by program DEMON using Mesh II.

![Profiles of subsidence](image)

**Fig. 4** Profiles of subsidence on the sloping ground surface predicted by program DEMON and the rays projection method

The subsidence predicted by the numerical modelling with Meshes I and II assuming elastic behaviour for the material with arbitrarily chosen deformational parameters may not be entirely realistic. However, as the emphasis is on the comparison of the subsidence patterns on the sloping ground surface predicted directly and by the rays projection method, therefore this aspect is ignored in the modelling.

**Maximum subsidence**

It can be seen from Fig. 4 that the maximum subsidence in profiles A, B, and C are close to each other and occur approximately above the centre of the extracted panel. It seems that for the surface inclination considered, the maximum subsidence does not, either in location or magnitude, appreciably alter compared with that for the
equivalent horizontal surface. However, as further discussed later, with the steepening of the ground surface, the magnitude of the maximum subsidence increases and its position moves toward down-slope.

Subsidence profiles

Compared with profile C, the subsidence values of both profiles A and B increase on the down-slope side and decrease on the up-slope side of the extracted panel, indicating the influence of the inclination of the ground surface on the shape of the subsidence profile. Generally speaking, profile B is closer to profile A, indicating the validity of the rays projection method using Eqs. (1) and (6) of predicting mining subsidence for sloping ground surfaces.

The results from the above study may be summarized as follows:

1. The ground inclination affects the distribution of subsidence. Compared with the subsidence on the equivalent horizontal surface, the magnitude of subsidence increases on the down-slope side and decreases on the up-slope side of the extraction.

2. The rays projection method can be used to determine subsidence effects on a sloping ground surface, especially for a surface with a long slope, from the subsidence on the equivalent horizontal surface, which could be predicted by any method, say, an empirical one.

EFFECTS OF GROUND INCLINATION ON SUBSIDENCE COMPONENTS

To further study the influence of the inclination of a ground surface on the patterns of subsidence movements, the profiles of subsidence, horizontal displacement and horizontal strain predicted for a given extracted panel at different ground inclination angles using the rays projection method are analyzed.

Eqs. (1), (4) and (5) together with Eqs. (6) and (7) are used to calculate subsidence $s(x)$, horizontal displacement $u(x)$ and horizontal strain $e(x)$ on the sloping ground surface from the corresponding components $s_d(x_d)$, $u_d(x_d)$ and $e_d(x_d)$ on the equivalent horizontal surface for inclination angles $0^\circ$, $15^\circ$, $30^\circ$ and $45^\circ$ respectively.

A panel of the extraction width $(W)$ of 210 m and mean extraction depth $(H)$ of 150 m is assumed, giving $W/H$ ratio equal to 1.4. A horizontal seam with the height of extraction $(M)$ of 2 m and an angle of draw $(\gamma)$ of $35^\circ$ is also assumed.

The subsidence, horizontal displacement and horizontal strain on an assumed equivalent horizontal surface are pre-calculated based on Subsidence Engineers' Handbook (National Coal Board. 1975). First, the maximum values of subsidence, tilt and horizontal strains (both tensile and compressive) on the equivalent horizontal surface are predicted. Since horizontal displacement is directly proportional to tilt, the maximum horizontal displacement is obtained next from the maximum tilt by multiplying a proportionality factor, which is assumed to be equal to 10 in the calculations (Shu and Bhattacharyya, 1990, 1992). Second, the complete profiles of subsidence and horizontal strains on the equivalent horizontal surface are predicted. The tilts are then calculated from the differential subsidence along the subsidence profile. Finally, the complete profile of horizontal displacement is obtained by multiplying the tilts by a factor of 10.

ANALYSIS OF THE RESULTS

The predicted magnitudes of the maximum subsidence, horizontal displacement and horizontal strains on the equivalent horizontal surface and the sloping surface for the inclination angles of $0^\circ$ (i.e. the equivalent horizontal surface), $15^\circ$, $30^\circ$ and $45^\circ$ are shown in Table 1. The profiles of subsidence, horizontal displacement and horizontal strain on the ground surface for the different inclination angles are presented in Figs. 5 to 8 respectively.

From the above results, following are observed.

Table 1

<table>
<thead>
<tr>
<th>Angle of Inclination</th>
<th>Maximum Subsidence ($m$)</th>
<th>Maximum Horizontal Displacement ($m$)</th>
<th>Maximum Tensile Stress ($m$)</th>
<th>Maximum Compressive Stress ($m$)</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Dip side</td>
<td>Rise side</td>
<td>Dip side</td>
<td>Rise side</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>1600</td>
<td>993</td>
<td>293</td>
<td>293</td>
</tr>
<tr>
<td>$15^\circ$</td>
<td>1634</td>
<td>992</td>
<td>215</td>
<td>11.9</td>
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<tr>
<td>$30^\circ$</td>
<td>1769</td>
<td>534</td>
<td>145</td>
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</tr>
<tr>
<td>$45^\circ$</td>
<td>1994</td>
<td>788</td>
<td>85</td>
<td>42.0</td>
</tr>
</tbody>
</table>
Fig. 5 Profiles of subsidence, horizontal displacement and horizontal strains on the equivalent horizontal surface

Fig. 6 Predicted profiles of subsidence, horizontal displacement and horizontal strains on a sloping ground surface with the angle of inclination of 15°

Fig. 7 Predicted profiles of subsidence, horizontal displacement and horizontal strains on a sloping ground surface with the angle of inclination of 30°

Fig. 8 Predicted profiles of subsidence, horizontal displacement and horizontal strains on a sloping ground surface with the angle of inclination of 45°

Maximum subsidence

The magnitude of the maximum subsidence slightly increases with the increase of the inclination angle. The position of the maximum subsidence moves down the slope from the point directly above the centre of the extracted panel where it is located when the ground surface is horizontal. The increase of the magnitude of the maximum subsidence and the shift of its position toward down-slope are slight when the inclination angle is small but become more pronounced with the steepening of the ground surface.

Maximum horizontal displacement and horizontal strain

As the inclination angle increases, the magnitudes of the maximum horizontal displacement and tensile strain dramatically increase on the down-slope side and decrease on the up-slope side of the extraction. Therefore, there are appreciable differences between the corresponding magnitudes of the maximum horizontal displacement and tensile strain on the down-slope and up-slope sides, and the differences increase as the surface slope steepens. A similar trend of change is found for the maximum compressive strain.

Fig. 9 shows the variation of the maximum horizontal tensile strain with the inclination of the ground surface obtained by using the rays projection method for a range of extraction width-depth ratios. It can be seen that with the increase of the angle of inclination, the maximum tensile strain for a given width-depth ratio increases on the
increases on the down-slope side and decreases on the up-slope side to a greater degree than that of the maximum subsidence. The results indicate that the sloping of the ground surface has more effect on the derivatives, especially the higher order derivatives (i.e., horizontal strain and curvature) of subsidence, than subsidence itself.

Profiles of subsidence, horizontal displacement and horizontal strain

All the profiles of subsidence, horizontal displacement and horizontal strain for a sloping ground are asymmetrical, and the asymmetry increases with the angle of inclination. More specifically, subsidence, horizontal displacement and horizontal strain on the down-slope side are always more pronounced than those on the up-slope side and the differences increase with increasing inclination angles.

However, the length of the ground surface affected by the undermining of the panel is longer on the up-slope side than that on the down-slope side and the difference increases with the angle of inclination. Correspondingly, the stretches of tension and compression zones, especially the former, are larger on the up-slope side than on the down-slope side of the extraction. With the increase of the angle of inclination, the differences increase.

**EFFECTS OF SURFACE TOPOGRAPHY ON MINING SUBSIDENCE OBSERVED BY OTHER RESEARCHERS**

Case histories about the effects of surface topography on mine subsidence have been reported from field observations by a number of researchers from several countries e.g. Gentry and Abel (1978), Adamek and Jeran (1983), Allgaier (1982), Conroy and Guzmary (1982), Fojes (1986), Tang and Peng (1986), Khair et al. (1987) and Peng et al. (1987) from the U.S.A.; Kapp (1982), Whitfield (1984), Pells et al. (1987), Hilleard (1988) and McNally (1989) from Australia; Fisekci et al. (1981) and Chrzanski et al. (1985) from Canada; Forrester and Whittaker (1976) and Whittaker and Reddish (1989) from the U.K. The reported case histories include a variety of topographical features such as steep slopes, rugged mountains, hills, valleys and flat bottom land. These
features, together with specific geological and mining conditions, complicate the subsidence effects and make the characteristics of subsidence different from one instance to another. The major findings from these case histories can be summarised as follows:

1. Surface topography does not seem to have much influence on subsidence (i.e. the vertical displacement) itself, but does have a significant effect on horizontal displacement and horizontal strain, especially tensile strain.
2. High tensile strains may develop along ridge lines, behind cliff faces and on steep slopes. Large compressive strains may be experienced at valley floors and flat bottom land.
3. Excessive tensile strains are likely to result in surface fractures and cracks and cause damages to structures on the ground surface.
4. Horizontal displacements, especially down-slope displacements, on a sloping ground surface can be significant and the direction of the displacement seems to be highly influenced by the topography.

Since few of the reported case histories seem to directly relate to extended sloping ground surfaces, it is difficult to make a quantitative comparison between the subsidence effects on such ground surfaces predicted in this paper and the cited observations of the effects of surface topography on mine subsidence. However, a qualitative comparison indicates that the results from the prediction generally agrees with those from the in-situ observations.

CONCLUSIONS

The subsidence effects on a sloping ground surface have been studied by using the finite element numerical modelling technique both directly and using a combination of an assumed equivalent horizontal surface and the rays projection method. The subsidence, horizontal displacements and horizontal strains from the mining of a given panel at the ground surface for different inclinations are compared using the rays projection method. The predicted subsidence effects are compared with the summarised findings from the field observations by other researchers. The following conclusions can be drawn from the study.

1. The profiles of subsidence and its components for a sloping ground are all asymmetrical, with the magnitudes being larger on the down-slope side than on the up-slope side of the mined-out area. However, the subsided zone is more extended at the up-slope side.
2. The inclination of a ground surface has greater effect on the horizontal movements and derivatives of subsidence, especially horizontal strains and curvatures.
3. Mining operations under sloping ground conditions can result in high tensile strains on the down-slope side of the mined-out area and extensive zones of low tensile strains on the up-slope side. These effects can cause the opening of joints and discontinuities and induce fractures and cracks on the ground surface.
4. Actual topographic features such as rugged mountains, steep hills, valleys and flat bottom land further complicate the subsidence effects due to underground mining operations. Large tensile strains may develop along ridge lines, behind cliff faces and on steep hills. High compressive strains may be experienced at the valley floors and flat bottom land.

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