EVALUATING COAL PILLAR MECHANICS THROUGH FIELD MEASUREMENTS

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ABSTRACT

Designing coal pillars to provide resistance against overburden loads has long been an aim of rock mechanics engineers. The need for accurate pillar strength models has become more urgent as greater overburden loads are encountered and pillar sizes grow larger. Current pillar design models differ widely in their predictions of the trend in pillar strength with increasing pillar width-to-height ratio. The goal of this paper is to evaluate current pillar strength theories, using a comprehensive data base of stress measurements from coal pillars. The stress measurements indicate that coal pillars maintain relatively high stresses near the ribline, and that the stress gradient within the yield zone can be approximated as a straight line. Several problems are identified for further research, including calibration procedures for different types of stress cells, strain softening behavior in the yield zone, and measurement of stresses in the cores of very wide pillars.

INTRODUCTION

The scientific study of coal pillars dates back at least 100 years. The first pillar strength formulas were derived from laboratory testing of coal specimens of different sizes and shapes (Bunting, 1911; Zern, 1926). Other measurement-based formulas incorporated results from large-scale compressive tests of coal specimens (Greenwald and Hartman, 1936; Bieniawski, 1964; Wang, 1977).

Another approach to pillar design relies on statistical analysis of previous experience with mine pillar performance. The first rigorous application of the method of back-analysis was by Salamon and Munro (1967) in South Africa. Sheory et al. (1987) recently employed a similar approach, combined with extensive laboratory and in situ testing, in developing a pillar strength formula for Indian coal mines.

These empirical methods have been validated through full-scale tests and many years of actual use. They contain considerable wisdom regarding coal pillar behavior. Their most significant drawback is that their predictions cannot be extrapolated with confidence beyond the data they were developed from, typically pillars with width/height ratios (w/h) of less than 5.

More fundamentally, empirical formulas treat the entire coal pillar as a single structural element. Their goal is to estimate the strength of the pillar, which is defined as the pillar's ultimate load-bearing capacity divided by its area. In reality, it has been known for some time that at ultimate load the stress within even a relatively small pillar is highly nonuniform. Tests conducted nearly 20 years ago by Wagner (1973) demonstrated this quite dramatically. Moreover, pillar failure is typically progressive, rather than sudden. Large sections of a pillar often yield (fail) long before the pillar as a whole attains its maximum load. Therefore, the concept of pillar strength, while useful for practical engineering design, is not very satisfactory from a scientific standpoint.

The mechanics-based approach to coal pillar design begins with the concept that a coal pillar can consist of an outer "yield zone" providing constraint to a "confined core". Perhaps the best-known of these mechanics-based methods was first proposed by A. H. Wilson (1972), and...
later revised (Wilson, 1983). Wilson's model assumed that coal follows a linear Mohr-Coulomb failure criterion. The vertical stress within the yield zone was then explicitly represented by an exponential vertical stress gradient. Wilson assumes that there is no further loss of strength or confinement once the coal has yielded, so the ultimate pillar resistance may be determined by integrating the vertical stress gradient over the entire area of the pillar. The pillar strength is simply the ultimate pillar resistance divided by the pillar area. Another formula, which implies an exponential rise in strength for squint pillars, was proposed by Salamon and Wagner (1985).

In part because an infinite exponential stress rise within the yield zone appears unrealistic, Barron (1984) presented an alternative formulation. His model limits the stress rise by two means: 1) a nonlinear Mohr envelope is assumed for the fracture strength of the intact coal, which allows the friction angle to gradually decrease, and 2) a "pseudo-plastic" failure mechanism is introduced at high stress, where the frictional strength exceeds the fracture strength. The result is that pillar stress grows at a decreasing rate, until it reaches a final, limiting value in the "pseudo-ductile" zone.

"Closed form" analytical solutions for pillar strength, like Wilson or Barron's, have, in recent years, increasingly been supplemented by numerical models. Like the analytical mechanics-based formulas, numerical models require failure criteria, a model of postfailure criteria, and material properties. The output from numerical models is typically a stress gradient that can be compared with Wilson's model. More sophisticated numerical models can consider roof and floor properties (Hsu and Peng, 1985), nonlinear failure criteria (Park and Gill, 1988), strain softening behavior (Kripakol, 1981; Zipf and Heasley, 1989), and geologic contacts (Inneschion, 1990).

As the models become more complex, more material properties are required. In situ values for material properties are seldom available. Modelers have employed either linear Mohr-Coulomb or nonlinear Mohr failure criteria, generally derived from laboratory strength tests. These criteria usually generate an exponential stress rise in the yield zone similar to those predicted by Wilson.

It is evident that a wide variety of techniques for predicting coal pillar strength are available to today's rock mechanics specialist. The predictions of these different formulas can vary widely, however. Three general trends are evident. The first, represented by the numerical models, Wilson's formula, Hsu-Peng, Kripakol, and Salamon's squat formula, show an exponential increase in pillar strength as pillar width is increased. These theories all assume that confinement grows rapidly within squat pillars, allowing the core stress to build indefinitely. The other outcome, represented by the Salamon-Munro and Holland-Gaddy empirical formulas and Barron's analytic formula, indicates that the pillar strength tapers towards some maximum, limiting value. A intermediate trend, an approximately linear increase in strength, is predicted by the empirical formulas of Bieniawski, Shoeney, and Gtart-Duval.

Figure 1 compares three formulas that are representative of these trends. They are Bieniawski's formula (equation 1), the Salamon-Munro formula (equation 2), and Wilson's formula (equation 3).

\[
S_x = \left( \frac{0.021 \times 0.999 \times \phi}{h} \right)
\]

\[
S_x = S_y \times \frac{w}{w}
\]

\[
S_x = \frac{1}{k} \times (\phi - \frac{1}{2}) \times P
\]

where:

- \(S_x\) = Pillar strength,
- \(S_y\) = In situ coal strength,
- \(w\) = Pillar width,
- \(h\) = Pillar height,
- \(k\) = triaxial stress factor,
- \(\phi\) = internal friction angle,
- \(P\) = unconfined compressive strength of the failed coal at the pillar edge.

In figure 1, the formulas were normalized to yield the same strength prediction for a width-to-height ratio of five.

The implications of the variation in pillar strength prediction, shown in figure 1 are enormous. As mines are worked at greater depth, and as mining methods like longwalls add...
to pillar loads, large pillars with $w/h$ of 10, 20, and even more are becoming the norm. A better definition of the trend in pillar strength has become a practical necessity if rock mechanics theory is to aid design practice.

The goal of this paper is to evaluate these three basic models of coal pillar strength. The basis of the evaluation will be a comprehensive data base of stress measurements from actual coal pillars.

**STRESS DISTRIBUTION PROFILES FROM PILLAR STRENGTH FORMULAS**

Since stress measurements are made at discrete points, they cannot be directly compared with pillar strength. Therefore, the first task is to develop stress distribution profiles from the various formulas. Stress gradients can be readily obtained from analytic formulas, such as Wilson's. Numerical models also provide stress distribution profiles, though not normally in the form of an equation. Stress gradients are not so easily obtained from empirical formulas, however. It was stated earlier that the empirical formulas do not explicitly consider the effect of internal pillar mechanics. It is apparent, however, that they imply a nonuniform stress distribution because of the shape effect.

Mark (1988) presented a method for determining the stress gradients that are implied by the empirical formulas. The derivation makes two assumptions that are implicit in Wilson's and other analytical formulations:

1. The stress within the yield zone of a given pillar is a continuous function of the distance from the rib ($z$) divided by pillar height ($h$), and independent of the pillar width ($w$).
2. The stress gradient within the yield zone of a given pillar does not change with time or load (i.e., the yielded coal is perfectly plastic).

The following equations show the stress gradient in the yield zone predicted by two sample empirical formulas using this derivation:

**BIENIAWSKI:**

$$\sigma_z = S_0 \left( 0.04 - 2.16 \frac{z}{h} \right)$$

**SALAMON-MUNRO:**

$$\sigma_z = S_0 \left( \frac{2.47 x^{0.88}}{h} \right)$$

**FIELD STUDIES**

During the past 20 years, a considerable effort has been directed towards studying pillar loading and pillar strength underground. Studies have been conducted by numerous research organizations, in every major U.S. coalfield, using several different types of instrumentation. At six of these mines, the coal pillars were loaded by some form of retreat mining and cells were installed in a pattern that allowed stress profiles to be constructed. The characteristics of the instruments used in the study are discussed, followed by brief descriptions of each of the study sites.

**Stress Measurement Techniques**

Three different types of instruments were

Figure 1. - Comparison of pillar strength predictions from various formulas.
used in the studies to measure pillar stress. These are the vibrating wire strainmeter (VWS), the borehole pressure cell (BPC), and the borehole platened flatjack (BPF). Each cell has its own characteristics, and its own calibration procedure, or procedure for relating cell response to rock stress or stress change.

The VWS is a "hard inclusion" device for measuring undirectional stress changes in rock. Introduced in the early 1970s (Hawkes and Bailey, 1973), the VWS consists essentially of a wire tensioned across a steel cylinder. As the rock stress changes, the cylinder deforms, causing the tension in the wire to change. Hawkes and Bailey's calibration procedures, with an intermediate value of the calibration coefficient, was used to reduce all the VWS data used in this study.

BPC have been used to monitor pressure changes in coal pillars for more than 25 years (Panak and Sturz, 1964). They consist of a hydraulic flatjack encapsulated in grout that is inserted into a cylindrical borehole. The BPF differs from the BPC primarily in that the grout is replaced by two aluminum pistons. Hydraulic cells are considered "soft inclusions," which complicates the development of calibration procedures. Although calibration of the BPC has been the subject of much discussion through the years (Babcock, 1988; Lu, 1984), much BPC data, including most of the data used in this study, is reported in terms of cell pressure rather than stress change. For BPF data, a calibration procedure developed by Heasley (1969) was used to reduce cell pressure to stress change.

Two issues may affect interpretation of pillar stress measurements. The first is the influence of the biaxial stress field. Biaxial stress analysis is simplified in coal pillars because the directions of the major and minor principal stresses can usually be assumed to be vertical and horizontal. In most of the studies described here, the stressmeters were oriented in the vertical direction only. Horizontal stress changes would still be expected to influence the response of vertically oriented stress cells. In the studies included in this paper, all the VWS data was adjusted using measured horizontal stress data, while the BPF and BPC data were not.

A second issue is the effect of coal failure on cell response. By definition, measurements of stresses in the yield zone are conducted where the coal is assumed to have failed. Yet very little research has addressed this issue, and all of the calibration procedures developed to date assume that the coal remains elastic. It may be added that measurements from within the yield zone, including those used in this study, usually appear reasonable and relatively consistent.

**Description of the Field Sites**

The Colorado School of Mines under a Bureau contract (Wang et al 1979) instrumented two coal pillars at the Keystone No. 1 Mine in McDowell County, WV. These two pillars were reduced in size from 24- to 24-m to 13- to 13-m to determine the in situ deformation characteristics and failure strength of coal. The Pocahontas No.3 Coalbed is 1.5 m thick and under 245 m of overburden at this site. Four stress profiles (1 to 4) shown in figure 2 are from the 19- to 19-m pillar and four other profiles (5 to 10) are from the 13- to 13-m pillar. All measurements were made with the VWS instrument.

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**Figure 2.** Yield zone stress profiles using the VWS at the Keystone No.1, Lynch No.37, and Kitt Mine.

U.S. Steel Research (Schauerger, 1985) instrumented four pairs of longwall pillars, the longwall panels, and the barrier pillars with more than 80 VWS. The Harlan Coalbed at this site is 3.4 m thick and under 475 m of overburden. Two stress profiles shown in figure 2 (11 & 12) are from a 22- by 28-m T1 gate entry pillar. The Pennsylvania State University instrumented four pillars at the Kitt Mine in Barbour County, WV with the VWS instrument (Mark, 1987). The Lower Kittanning Coalbed averages 1.5 m thick and is under approximately 210 m of overburden at these two sites. Two
stress profiles (13 & 14) shown in figure 2 are from two adjacent 13- by 22-m gate entry pillars, and two other profiles (15 & 16) are from a 7- by 34-m gate entry pillar.

In the late 1980's, the U.S. Bureau of Mines instrumented 12 different pillars within three longwall gate entry of the VP No. 3 Mine near Vanwall, VA (Jannaccone, 1988 and Campoli et al, 1990). The Pocahontas No. 3 Coalbed averages 1.7 m in thickness with a overburden ranging from 545 to 625 m over the three instrumented sites. One stress profile (1) shown in figure 3 is from a 24- by 24-m 7D gate entry pillar, three stress profiles (2 to 4) are from a 37- with the BPF instrument.

The Bureau (DeMarco et al., 1988) and Agapito and Associates (Maleki and Moon, 1988) both instrumented several 3 m thick pillars within the Wettis Coalbed at the Plateau Mine near Price, UT. All four stress profiles shown in figure 4 were from two 9- by 27-m gate entry pillars under 425 m (1 & 2) and 460 m (3 & 4) of overburden. All measurements were made with the BPC instrument.

The Bureau has monitored six different sites along three gate entries of the Foide Creek Mine near Oak Creek, CO (Hann et al., 1991). The Wedge Coalbed averages 3 m thick while the overburden averages 320 m over the entire study area. Two stress profiles (5 & 6) shown in figure 4 are from the 12- by 21-m 5L-74 gate entry pillars, three other profiles (7 to 9) are from the 12- by 21-m 5L-74 gate entry pillars, one profile (10) is from the 12- by 21-m

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**KEY**

- Foide Creek Mine, profiles 1 to 8
- Plateau Mine, profiles 9 to 12

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**Figure 3.** Yield zone stress profiles using the BPC at the Plateau and Foide Creek Mines.

by 55-m 8D gate entry pillar, and two stress profiles (5 & 6) are from another 37- by 55-m 10D gate entry pillar. All measurements were made

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**Figure 4.** Yield zone stress profiles using the BPC at the VP No. 3 Mine.

5L-53 gate entry, and two profiles (11 & 12) are from the 12- by 21-m 5L-74 gate entry. All measurements were made with the BPC instrument.

**Selection of Stress Profiles**

As the goal of the study is to evaluate formulas predicting the stress rise within the yield zone, a key task was the selection of stress profiles from the field data. A typical stress profile is shown in figure 5. The profiles were identified using the following criteria.
Table 3 - Derived stress gradient parameters.

<table>
<thead>
<tr>
<th>Cell type</th>
<th>$\sigma_x$ @ x=0, MPa</th>
<th>$\Delta \sigma_x$, MPa/m</th>
<th>$\Delta \sigma_y$, MPa</th>
<th>$\sigma_x$ @ x/h=1, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>VWS</td>
<td>-2.3</td>
<td>6.2</td>
<td>6.3</td>
<td>10.9</td>
</tr>
<tr>
<td>BPF</td>
<td>-14.3</td>
<td>18.3</td>
<td>17.2</td>
<td>28.9</td>
</tr>
<tr>
<td>BPC</td>
<td>-3.5</td>
<td>11.7</td>
<td>7.1</td>
<td>21.6</td>
</tr>
</tbody>
</table>

$\sigma_x$ = vertical stress, $\Delta \sigma_x$ = change in vertical stress, x = distance from rib, h = pillar height, x = mean, s = standard deviation.

1. The measurements indicated that the pillar reached its ultimate resistance, or at least that pillar failure extended a significant distance into the pillar.
2. Only cells that appeared to be giving valid readings were used in the profiles.

From these 15 instrumented pillars, 34 stress gradients were used in this analysis (figs. 2 to 4). An average of 3.1 streamers were used in the construction of each profile. While most of the profiles extend less than two seam heights (2h) into the rib, the deepest profile is 5.5h. and nine other profiles are more than 2.5h deep.

The biggest problem encountered in choosing the profiles is that they were seldom static. In general, as a pillar's load increased, and the yield zone expanded, the stresses within the yield zone tended to decrease (see fig. 5). This "strain softening" behavior within the yield zone may account for some of the scatter in the data.

ANALYSIS OF MEASURED STRESS GRADIENTS

The first task was to look at the shape of the stress profiles. Because of the limited number of streamers in each profile, linear regression was the only curve-fitting function applied to the data. The R-squared values obtained from the regression averaged 0.94 for three-cell profiles and 0.89 for four- and five-cell profiles. These results indicate that a linear model is probably appropriate for this data.

The magnitudes of the slopes, or stress gradients, were evaluated next. Two stress gradients were obtained for each profile, the first being the rate of stress rise per foot of distance from the rib ($\Delta \sigma_x$, MPa/m), and the second being the stress rise per seam height of distance from the rib ($\Delta \sigma_y$, MPa/h). The stress gradients were also divided into three categories by cell type.

The results, summarized in table 1, indicate that the VWS data yielded the lowest stress gradients, and the BPF data the greatest, for both types of gradient (MPa/m and MPa/h). Stress gradients from the BPC data were similar to those obtained from the VWS when evaluated as MPa/m, but approached those of the BPF when calculated as MPa/h. Table 1 also indicates that there is considerable scatter within each group of measurements. The standard deviation of the slopes was approximately 50% of the mean for all three cell types.

Multiple regression was employed to examine the effects of other parameters, including depth of cover, seam height, coal strength, and roof rock quality on the stress gradient. No meaningful correlations were observed when each cell type was analyzed separately. With only one to three study mines for each cell type, however, the range of each variable was quite limited. Indeed, the overwhelming influence of cell type may have masked effects due to the other parameters.

Figure 5 - Typical pillar stress profiles measured in the field. Stress measurements made at the VP No.3 Mine in a pillar with a w/h = 13.

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Further evidence of strain softening can be found in the data. Of the 10 stress profiles obtained at the Keystone Mine, four extended to a depth of 4.6 m or more. The average gradient for these profiles was 5.6 MPa/m, against 9.7 MPa/m for the 3 m profiles. Similar, though less pronounced, trends can be observed in the VT No. 3 and KIt Mine profiles.

The linear regression results were also used to evaluate the level of stress that is maintained near the rib. No actual measurements were made close to the rib, so the x=0 intercepts must be considered hypothetical. Also, in the case of the VWS and BPF data, the development stresses present before the measurements began must be included. Using the pillar and entry dimensions and the depth of cover at each site, the development stresses were estimated using tributary area theory. Tributary area theory was considered appropriate because local extraction rates were below 10% in all cases but two, and because the subsequent stress measurements indicated that no significant pillar yielding had occurred. The estimated total vertical stress at a distance of x=h from the rib was calculated as the development stress, plus the intercept, plus the stress increase due to the stress gradient. These data are summarized by cell type in Table 1. They indicate that significant vertical stresses are maintained at a distance of one seam height from the rib: approximately 17 MPa for VWS and BPF data, and 30 MPa for BPF data.

**PILLAR STRESS AND PILLAR STRENGTH FORMULAS FROM THE FIELD DATA**

The field data may be used to deduce pillar stress and pillar strength formulas that can be compared directly with the models. Using the data from Figures 2 to 4, and the estimated development loads (VWS and BPF data only), the estimated pillar stress gradients are

\[
\text{VWS} = 7.6 + 10.8 \frac{x}{h} \\
\text{BPF} = 4.1 + 28.3 \frac{x}{h} \\
\text{BPC} = 3.6 + 21.8 \frac{x}{h} \\
\]

These stress gradients imply pillar strength as

\[
\text{VWS} = 9.8 \left(0.9 + 2.8 \frac{x}{h}\right) \\
\text{BPF} = 8.9 \left(4.6 + 0.54 \frac{x}{h}\right) \\
\text{BPC} = 3.8 \left(1.2 + 0.8 \frac{x}{h}\right) \\
\text{Avg.} = 6.1 \left(0.44 + 0.58 \frac{x}{h}\right)
\]

Figure 6 shows the pillar stress gradients generated from equations 6 to 9, and Figure 7 compares the pillar strength gradients with three empirical models discussed earlier. For narrow
pillars, the VWS and BPF measurements predict pillar strengths significantly greater than any of the current models. For more squat pillars, the BPC and BPF data predict strengths exceeding any of the empirical formulas. The rate of stress increase is still much less than that predicted by an analytical formula, however.

These formulas are representative of measured stresses to \(x/h = 4\), and therefore apply to pillars of \(w/h = 8\). The only study sites at which a serious attempt was made to collect data within failed coal pillars of \(w/h > 8\) was at VP No. 3.

Although five such pillars have been instrumented, the data have been ambiguous. Figure 5 shows typical results obtained from an 24 m \((w/h = 13)\) pillar up until it was destroyed by a series of small rib bumps. In this case, as in the others, no stress changes were measured in the core that exceeded the peak stresses measured at approximately \(x/h = 4\). One explanation is that the peak stress attained by a coal pillar is limited by some physical factor, such as interface slip or the "pseudo ductile" failure described by Barron. Another possibility is that the BPF cells fail at stresses exceeding 69 MPa, which corresponds to gage pressures of more than 170 MPa.

If the conservative interpretation that pillar stresses cannot exceed the level attained at \(x = 4h\) is accepted, then the pillar strength may be adapted accordingly. Using the average stress gradient derived above (equation 13) for illustration, a formula for the strength of pillars in excess of \(w/h = 8\) is

\[
\frac{(65w^2 - 655w + 1740 + w^3)}{w^2}
\]  

This formula is also shown on figure 7.

**CONCLUSIONS**

1. There is currently a wide range of opinion regarding the stress within yielded coal pillars. One school of thought holds that the stress increases exponentially, while another believes the stress approaches an asymptotic maximum. Various intermediate interpretations are also current.

2. Empirical pillar strength formulae may be converted to pillar stress formulas, using the concept that the pillar load-bearing capacity is the pillar stress function integrated over the load-bearing area.

3. Stress measurements are available from many pillars for \(x/h\) up to 4. The measurements display considerable variability, much of it apparently attributable to the different cell types and calibration procedures that have been used.

4. The field data generally support the interpretation that stress gradient within the yield zone is approximately linear, and that relatively high stresses may be maintained near the ribline.

5. "Pillar strength formulas" may be derived from the field data. These generally predict higher strengths than current empirical formulas, but lower core stresses than most numerical models.

6. Little field data are available beyond a

![Figure 7 - Pillar strength formulas obtained from stress measurements compared with existing formulas.](image)

Distance of \(x = 4h\) from the ribline. What little there is, indicates that pillar stress may cease to increase. Other "pillar strength formulas" can be developed assuming the pillar stress remains constant beyond \(x = 4h\).

7. A number of areas for future research were identified:

a) Better calibration procedures for stress cells, including the effects of biaxial stresses and coal yielding, b) stress measurements within the cores.
of very squat pillars (w/h > 6), c) the effects of strain softening and time on pillar strength, and d) the effects of interfaces, seam height, depth, and material properties.

REFERENCES


11th International Conference on Ground Control in Mining, The University of New South Wales, N.S.W., July 1992.