DEVELOPMENT OF VOID DIFFUSION MODELS TO PREDICT SUBSIDENCE DUE TO UNDERGROUND COAL MINING

By

Qing-Wang Hao¹ and Yoginder P. Chugh¹

ABSTRACT

A generalized void diffusion model for subsidence prediction has been developed based on theoretical concepts and physical principles. The general form of the model is a three-dimensional differential equation. With a few assumptions on the characteristics of overburden strata overlying typical room-and-pillar mining or longwall mining systems, specific solutions for the differential equation were obtained. A brief description of these solutions and their applications in the United States and China are presented in this paper.

INTRODUCTION

Since the development of the void diffusion model (VDM) in China in 1988 (Hao, 1988a, 1988b), its application in the midwestern United States has been explored in the past three years. Modifications have been made in the VDM to consider specific problems such as the room-and-pillar mining which was not considered in the original development. The basic concepts of the void diffusion model, recent developments, and the up-to-date achievements of the relevant studies are presented in the following sections.

CONCEPTS OF VOID DIFFUSION MODEL

Ground movements due to underground mining can be described as a process of void migration and void diffusion. Voids which are created in an extraction area are subdivided into two parts. One part is absorbed by the building (volume increase) area, and the other part migrates to the ground surface as subsidence area. The unit which creates voids such as the extraction area is called the positive void source, while the unit which absorbs voids like the building area is called the negative void source. In order to quantify a void source, the concept of "void source intensity" is introduced. It basically means the void volume created or absorbed in a unit area during the considered period. Void creation or void absorption can be time-dependent or time-independent.

With the increase in height above the void source, the void passing or extension range in the horizontal plane gradually expands outward, which basically determines the characteristics of the subsidence profile for specific void source intensity distributions. This process is referred to as "void diffusion". With this definition, the void diffusion problem is similar to a time-dependent general diffusion problem such as heat transfer, water diffusion, underground water permeation etc. In this case, the vertical axis acts like a time axis since it is irreversible. The direction of void diffusion is defined as the general trend of void migration in the horizontal direction.

Based on three physical principles - conservation of void volume, linear void diffusion, and linear void convection, the general equation of void diffusion with convection has been established as below (Hao, 1988a, 1988b; Hao and Ma, 1990):

\[ \frac{\partial S}{\partial t} + \frac{\partial}{\partial x} (B_1 \frac{\partial S}{\partial x}) + \frac{\partial}{\partial y} (B_2 \frac{\partial S}{\partial y}) + \frac{\partial}{\partial z} (B_3 \frac{\partial S}{\partial z}) = 0 \]

where, \( B_1 \), \( B_2 \), and \( B_3 \) are the principal coefficients of void diffusion, \( A_1 \), and \( A_2 \) are the principal components of void convection, \( I(x,y,z) \) is the intensity of void source, \( S \) is the vertical subsidence of ground surface, \( x \) and \( y \) are the horizontal coordinates, and \( z \) is the vertical coordinate with positive direction upwards.

With the above governing equation, surface subsidence can be predicted with consideration of different material behavior. The following is a brief summary of the studies which have been done by the authors. Since void...
diffusion occurs in horizontal directions, the following discussions concerns material behavior in the horizontal directions.

**SUBSIDENCE PREDICTION IN ROOM-AND-PILLAR MINING: LINEAR HOMOGENEOUS MODEL.**

In room-and-pillar mining, the overburden strata generally do not fracture as in the longwall mining and the strata properties remain nearly unchanged before and after mining. It is, therefore, reasonable to assume that the void diffusion coefficient and the convection rate are constant in a horizontal plane. A general three-dimensional solution with an arbitrary distribution of void source intensity has been developed (Hao, 1988a, 1988b). In the case of a flat seam, the solution can be presented as below:

\[
S_{i} = \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} \frac{e^{-r_{i}^{2} - \sigma_{i}^{2}/2}}{4\pi \sigma_{i}^{2}} \, dx \, dy \, dz
\]

(2)

where

\[
r_{i} = \frac{\pi}{\sqrt{2\pi} \sigma_{i}}
\]

(3)

The key problem is to determine the void source intensity function (VSIF) \( f(x, y, z) \). In room and pillar mining, the volume change of openings can be treated as the void source. Since the mining thickness and opening size is small relative to mining depth, it is appropriate to consider the entire extracted area as a plane-shaped void source. The physical meaning of this type of void source intensity is the roof settlement. The following approach has been developed to determine the VSIF in an irregular and multi-panel mining system.

1. Divide the panels into several zones of constant extraction ratio.

2. For any point at the mining level, there is an influence circle with its center at that point. This circle may cover several zones with different extraction ratios. A weighted mean value of the extraction ratios with the circle is assigned to that point, which is called the virtual extraction ratio of that point \( E_{v} \).

![Diagram](image-url)

**Figure 1 Layout of Mine Workings, Monitoring Lines and LHVDM Grid Model**

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Figure 2  LHVDM Subsidence, Slope and Curvature Profiles and Comparisons along Line AB
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Figure 3 LHVDM Subsidence, Slope and Curvature Profiles and Comparisons along Line CD

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The void source intensity (VSI) at this point is correlated with the virtual extraction ratio of the point, i.e., 

\[ VSI = F(E_0) \]

To better represent the influence of each extraction unit on the concerning point, the distance between the point and the extraction unit can be taken into account through an influence function \( p(r) \). In our analysis the following influence function was used and verified to be valid:

\[ p(r) = \frac{1}{R^2} e^{-\frac{r^2}{R^2}} \]

where \( r \) is the distance and \( R \) is the radius of influence. Then the virtual extraction ratio can be calculated with the consideration of time-dependency as follows:

\[ E_i(t) = \sum_{i=1}^{N} E_i(t) \int_{A_i} p(r) \, dr \]

where \( E_i \) is the extraction ratio of zone \( i \), \( N \) is the number of zones with different extraction ratios, \( T(t) \) is the time coefficient.

A numerical algorithm has been developed based on the above discussion as follows:

1. Determine the surface area (SA) in which subsidence prediction is needed.
2. Determine the mined-out area (MA) which has influence on SA.
3. Divide MA into several zones based on the extraction ratio and time-dependency. Constant extraction ratio and time coefficient are assigned to each zone.
4. Draw a uniform grid around the MA with a grid interval of about 0.2r (as shown in Equation 3).
5. Calculate the virtual extraction ratio for all nodes of the grid.
6. Calculate the void source intensity value at each node.
7. Calculate surface subsidence.

A case study with pillar extraction has been accomplished using the above algorithm. The panel layout, subsideance monitoring lines, and the model are shown in Figure 1. Based on the field observation, the void source intensity function and time coefficient function were obtained. VSI can be calculated by

\[ VSI = 0.7 M e^{-\frac{r^2}{z_2^2}} \]

where \( M \) is the mining thickness. For the case of immediate subsidence following pillar extraction, the time coefficient \( T(t) \) can be expressed as:

\[ T(t) = 1 - e^{-at} \]

where \( a \) and \( b \) are constants. In our case, \( a = 1.5 \) and \( b = 0.125 \).

Figure 2 and 3 show the comparisons of predicted subsidence, slope and curvature profiles with the observed data. It can be seen that for such a complex mining system, the results are very encouraging.

LONGWALL MINING: NON-HOMOGENEOUS MODEL

In longwall mining, the overburden fracturing has an important effect on the characteristics of surface subsidence. Therefore, we think that the overburden fracturing should be taken into account for subsidence prediction. A simple two-dimensional model which accounts for the unbalanced disturbances over mined and unmined areas can be described as follows (Hao, 1988b; Hao and Chugh, 1990):

1. The overburden strata is subdivided into two parts with a vertical interface (\( x=0 \)). Unequal void diffusivity coefficients (\( B_1 \) and \( B_2 \)) are assumed for each side of the interface due to the unbalanced disturbances.
2. The void diffusivity intensity distribution is assumed as:

\[ J(x, y) = \begin{cases} \frac{0}{aM} & x<0 \\ \frac{aM}{bM} & x>0 \end{cases} \]

where \( M \) is the mining thickness, \( a \) is an empirical constant, and \( \delta(x) \) is the Dirac function. The above assumptions result in the following subsidence profile:

\[ \delta = \begin{cases} \frac{r_1^2 - r_2^2}{r_1^2 + r_2^2} [1 + \text{erf}(\sqrt{\frac{r_1}{z_2}})] & x<0 \\ \frac{r_1^2 - r_2^2}{r_1^2 + r_2^2} & x=0 \\ \frac{1}{z_{max}} \delta_{max} e^{-\frac{x^2}{z_2^2}} & x>0 \end{cases} \]

where
Figure 4 NHVDM Subsidence Profile and Comparisons over a Longwall Panel

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\[ r_1 = \frac{2}{\sqrt{\pi}} B \cdot d \]  
\[ r_2 = \frac{2}{\sqrt{\pi}} B \cdot d \]  

\[ \text{H is the mining depth, } \epsilon \text{ is the error function, and } \text{erf} \text{ is the error function.} \]

Obviously, if \( \epsilon = \text{erf} \), the subsidence profile given by Equation (9) becomes the well-known error function subsidence profile:

\[ s(x) = \frac{S_{\text{max}}}{r} [1 - \text{erf}(\sqrt{\frac{x^2}{r^2}})] \]  

Therefore, the NLVDM is a convenient way to construct a general error function subsidence profile profile. Figure 4 shows the comparison between the theoretical profiles and the observed data. The NLVDM profile is much closer to the observed data than the other empirical profiles, particularly around the trough edge areas.

**LONGWALL MINING: NONLINEAR MODEL**

Longwall mining subsidence can be treated by a more sophisticated approach - nonlinear void diffusion model (NLVDM). This model is solved with the aid of finite element method (FEM). The FEM equation is as follows (Hao, 1988b; Hao et al., 1991):

\[ K \{\delta\} + D \{\delta\} = R \]  

where \( \{\delta\} \) is the node displacement vector, \( K \) is the stiffness matrix, \( D \) is the time matrix, and \( R \) is the vector of void source intensity. These matrices are assembled from the interpolation functions of isoparametric elements and the distribution of void source intensity. In the case of flat lying strata, analysis can be conducted by the ADINA-T program since the void convection rate can be considered to be zero. The relationship between the void diffusion coefficient and the subsidence value can be determined from the field data. Figure 5 shows two examples. A subsidence profile in the Chinese coal field was predicted based on the above approach. For comparison, some of the known profile functions were curve-fitted to the observed data. To keep it consistent, all the slope and curvature values were calculated from the subsidence profile using the finite difference method. The results are shown in Figure 6. From these results, the following comments can be made:

1. Although the NLVDM profiles were predicted based on an average void diffusion coefficient (B), the others were fitted directly to the observed data, their accuracies are still very competitive.

2. Around the subsidence trough edge area, the NLVDM model fits very well to the observed subsidence data, and suggests a higher accuracy than the other empirical profile functions.

3. The NLVDM model predicts smaller values for the maximum slope and maximum curvature. But it provides a better fit to the observed slope and curvature data around the trough edge area.

In longwall mining, the trough edge areas are of most concern from the point of view of structural damage but are most difficult to predict with the empirical methods. The NLVDM model may solve this problem to some extent.

**CONCLUSIONS**

The void diffusion model (VDM) can predict subsidence with considerations of different mining and geologic conditions. It overcomes some of the deficiencies of conventional prediction methods such as profile function method and influence function method and gives better results.

**REFERENCES**


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(a) A Chinese Coal Mine

(b) A United States Coal Mine

Figure 5 Nonlinear Characteristic Curves of Void Diffusion Coefficient
Figure 6: NLVDM Subsidence, Slope and Curvature Profiles and Comparisons