APPLICATION OF A SIMPLIFIED THREE-DIMENSIONAL ROOF-PILLAR-FLOOR INTERACTION ANALYSIS MODEL FOR SUBSIDENCE PREDICTION

By

Witold M. Pytel¹, Yoginder P. Chugh⁴

ABSTRACT

A simplified, three-dimensional, time-dependent, mechanistic model of the overburden-coal seam-floor interaction has been developed and validated at two Illinois coal mines. The model-based on the thin plate theory-can involve any complex layout of mine workings, time-dependent sequence of extraction and rheological properties of immediate floor strata. The model output data consist of spatially distributed surface subsidence and its slope and curvature, as well as load acting on pillars in one or more panels. A comparison of calculated and monitored surface subsidence or in-mine convergence measurements has proven the model's effectiveness for surface subsidence and load transfer prediction.

INTRODUCTION

Time-dependent deformation of partial extraction mine layouts has gained engineering importance where coal seams are associated with relatively thick weak floor strata. Time-dependent deformation of weak floor strata results in pillar and roof settlement and corresponding floor heave in openings, surface subsidence and load transfer to adjoining mine workings. Therefore, the ability to predict the magnitude of such subsidence and load transfer becomes very important from the mine planning and safety of surface structures point of view. Toward this objective, a simplified three-dimensional roof-pillar-floor interaction model based on the theory of plates on inelastic foundations was developed (Pytel and Chugh, 1980, 1991a). This analytical technique permits consideration of:

1. three-dimensional mining geometries of several hundred pillars in a panel with time-varying asymmetrical mine geometry,
2. variable geology of overburden and floor strata associated with a coal seam,
3. different constitutive behavior for different coal measure rocks, and
4. presence of mined-out areas.

The analytical problem solution is based on the finite difference method which permits one to relate the actual mining progress with equivalent time-dependent overburden/floor deformability, using an incremental advance approach, which can consider all mined-out areas, with different times of extraction. Pytel and Chugh (1991a) utilized the model to perform sensitivity analyses on selected mine system variables such as time-dependent behavior of overburden and floor strata, and elastoplastic behavior of coal seam. From a practical point of view, rheological aspect of floor strata behavior was found to be most important for shallow room-and-pillar mining. The model was validated at two of the central Illinois mines, for the long-term surface subsidence movements associated with weak floor strata. Time-dependent properties of floor strata were estimated from in-mine convergence measurements and observed surface subsidence.

MODEL DESCRIPTION

INTRODUCTION

The theoretical background of the model has been discussed in detail elsewhere (Pytel and Chugh, 1991a); a brief summary is provided here for the sake of completeness. The physical problem consisting of overburden plate, coal seam strata unmined or in the form of room-and-pillar mining panels, and floor strata, is transformed into an equivalent mechanics problem after dividing the system into smaller blocks through a rectangular grid network depending upon the required solution accuracy. Each block may be assigned different coal measure rock properties and loading conditions. The overburden associated with the coal seam in each block is transformed into a composite plate with stepwise varying flexural stiffness B with depth. The uniformly distributed overburden load is

1. Senior Scientist, Department of Mining Engineering, Southern Illinois University at Carbondale, Illinois, U.S.A.
2. Professor and Chair, Department of Mining Engineering, Southern Illinois University at Carbondale, Illinois, U.S.A.

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transmitted to the weak floor strata through segmented rectangular footings representing panel pillars. The contact stresses at the soil/rock plate interface are approximated by rectangular areas of uniform stresses which are transformed into equivalent concentrated forces acting at the center of the plan area of each element. Coal pillars are represented by a set of nonlinear springs sandwiched between the upper overburden strata plate and the lower deformable weak floor strata. The model is based on the theory of flexure of thin plates. Therefore, the overburden strata thickness should be small as compared to the width of panels including barrier pillars, and the ratio of expected surface subsidence to overburden thickness should also be small. Additional assumptions to simplify the complexity of this particular model are given below.

1. Rheological constants of the materials are stress independent.
2. Poisson's ratio of material is independent of time and stress.
3. Smooth coal roof and coal floor interfaces are required.
4. Only vertical external load may be included in the analyses.

The immediate weak floor strata are transformed into an equivalent single rock layer with uniform time-independent and time-dependent deformability parameters resting on an undeformable rock mass. The broad comparative analysis of the subsidence models applicable in this case was presented elsewhere (Chugh et al., 1993). For typical U.S. coals conditions, where the ratio of weak floor strata thickness H and pillar width B is smaller than 0.2, the one-parameter linear model valid for compressive loading conditions with no lateral strain may be utilized.

**Calculation of Overburden Plate Deflection and Load Distribution**

The biharmonic flexural equation (Eq. 1), with its boundary condition equations, is replaced by a set of equivalent algebraic

\[ D \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \delta(x,y) + \delta(x,y) = q(x,y) \]

equations and solved using a finite difference approach. The deflection of the overburden, assumed to be a thin plate resting on the deformable layer, is obtained from the displacements of an array of blocks. The determined plate deflection matrix is the basis for calculation of the surface slope and curvature as well as all internal loads within the overburden plate.

The overburden plate is assumed to be rigidly clamped along its edges over the intact zone. Currently, the maximum grid network size which may be considered is 200x200 pillars. However, the size of the network considered depends upon the required solution accuracy as well as upon the size of the computer.

**TIME-DEPENDENT MODEL ANALYSIS**

**Calculation of Time-Dependent Surface Subsidence**

Time-dependent deflection of the overburden plate was investigated based on rheological properties of floor strata. Since a truly time-dependent approach requires consideration of the time varying development of mine workings, the problem must be solved using an incremental mining sequence. Therefore, all mine workings developed in the total period of time \( t_b \) are divided into \( b \) phases with an assigned extraction time \( t_i \) for each of the phases, and a suitable function of the load history \( H(t) \).

Time-dependent subsidence at point \( (x, y) \) after time \( t_b \) may be expressed as follows:

\[ w(x, y, t_b) = \sum_{i=1}^{b} W(x, y, t_i) \]

where \( W(x, y, t_i) \) is the \( i \)-th phase subsidence after \( i \)-th phase of extraction and \( t_i \) is the time of extraction

\[ t_i \text{, and after time } t_j = \sum_{i=1}^{j} t_i \]

**Estimation of Model Parameters**

The transformation (creep) function equations for floor strata and their derivation were presented recently (Pytel and Chugh, 1989). The function \( f(t) \) satisfies the following differential equation:

\[ f(t) + \int f(t) \exp \left( \frac{t - t'}{N_1} \right) dt' \leq E_1 \]

where \( E_1, E_2, N_1, N_2 \) are elastic and viscous parameters of the Kelvin-Voigt and Maxwell units, respectively. Assuming a linear hereditary model for weak floor strata behavior, the time-dependent deformability modulus of weak floor strata may be expressed as \( E(t) = E_2 - E_1 t \).

The model parameters \( E_1, E_2, N_1, N_2 \) may be estimated using the approaches listed below.
Plate loading test (incremental) time-dependent plate loading tests at constant load may be conducted to obtain time-dependent deformation parameters of weak floor strata. The data can then be analyzed to estimate $E_1$, $E_2$, $N_1$, and $N_2$ parameters using the available numerical procedures, e.g., the least squares method, or other simpler techniques (Chen and Chugh, 1990). Since plate loading tests, however, cannot be conducted over a long period of time, the $E_1$, $N_1$, and $N_2$ parameter values reflect only short-term behavior of weak floor strata. The parameter $E_2$, representing initial elastic floor deformability, may also be calculated from the approximate statistical relationship (Paldia et al., 1990):$$E_2 = 300.1 \left( \frac{MC}{0.47} \right), \text{ [MPa]}$$where $MC$ is the average moisture content [%] of weak floor strata down to a depth of about 0.45 m below the coal seam.

In mine convergence measurement These are very suitable for the determination of long-term values of $E_1$, $N_1$, and $N_2$ parameters. The parameter $E_2$ must still be estimated from plate loading test data or using other available techniques, e.g., Eq. 4. It is important to note that the time-dependent parameters determined from convergence measurements represent overall rock mass behavior associated with the opening and pillars rather than weak floor strata only. These parameters are truly representative for the physical problem in the authors opinion. Roof-floor convergence monitoring, however, does not give directly the magnitude of the pillar settlement which is the basis for estimation of time-dependent parameters. The ratio of convergence to adjacent pillar settlement depends on several parameters, among which the Poisson's ratio $v$, relative weak floor strata thickness $H/B$, and extraction ratio $e_a$ (or ratio of pillar size to opening width $B/W$) are the most important. Based on the theory of elasticity, the plane problem of punching of rigid pillar of size $B$ into a deformable rock layer of thickness $H$ was solved using the finite difference method (Pytel and Chugh, 1991b). The ratio $e_a$ of floor heave $h_p$, average floor surface vertical displacement within the opening to the adjacent pillar settlement $Y_p$ calculated for average Illinois conditions ($v=0.35$) is presented in Figure 1. Therefore, assuming no vacuum behavior of coal and no time-dependent variation of the external load acting upon the pillar, the actual time-dependent pillar settlement based on a given rheological model characterization may be estimated from the monitored roof-floor convergence data $C$ as follows:

$Y_p = C/(1+e_a)$ \hspace{1cm} (9)

where $e_a$ is the coefficient taken from Figure 1.

The surface settlement determined directly or estimated from roof-floor convergence data may be utilized for estimation of time-dependent parameters of the weak floor strata using the least squares method. The average ratios of the $E_1$, $E_2$, $N_1$, $N_2$, values of the Burgers' model parameters, recently obtained from several Illinois mines using the above approach, are given below:

$E_2/E_1 = 1.08, E_2/N_1 = 0.041 \text{ [1/day]},$ and $E_2/N_2 = 0.005 \text{ [1/day]}$.

These may be utilized for preliminary analyses in Illinois if site-specific accurate data are not available.

MODEL VALIDATION

MINE I

Mine description and area geology

The three-dimensional mine layout consisting of three panels (Figure 2) was analyzed and the calculated subsidence data were compared with the monitored corresponding time-dependent surface displacements. The average mine geometry and geotechnical parameters used in the analysis are given below.
Fig. 2 - Layout of mine workings at Mine 1 and subdivision of the mined-out areas into phases and zones for subsidence prediction.

1. overburden depth: \( H = 87.3 \text{ m} \)
2. overburden pressure before mining: \( Q_v = 2.17 \text{ MPa} \)
3. Poisson's ratio for weak floor strata: \( v = 0.35 \)
4. average mining height: \( H = 1.77 \text{ m} \)
5. modulus of elasticity for coal: \( E = 1.3393 \text{ MPa} \)
6. opening and pillar sizes in different parts of the mine are shown in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Location</th>
<th>Opening width</th>
<th>Solid pillar width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main entries</td>
<td>4.67</td>
<td>25.69</td>
</tr>
<tr>
<td>Sub-main entries</td>
<td>4.87</td>
<td>22.55</td>
</tr>
<tr>
<td>Panela</td>
<td>4.10</td>
<td>14.76</td>
</tr>
</tbody>
</table>

Overburden stiffness characterization

The overburden cross-section, provided by the company, is shown in Table 2. The overburden was transformed into a composite plate with flexural stiffness \( D \) given below and calculated for two extremal cases.

1. fully bonded layers: \( D = 4.5667 \text{ MN m}^2/\text{cm} \)
   (this value was utilized in the analysis)
2. perfectly smooth interfaces between layers:
   \( D = 4.1066 \text{ MN m}^2/\text{cm} \)

Weak floor strata characterization

The thickness \( H \) of the weak floor strata and its moisture content \( MC \) were available from two sites (Site 1 and Site 2) at this mine.

**Site 1**
1. thickness of weak floor stratum:
   \( H = 1.27 \text{ m} \)
2. moisture content (%):
   \( MC = 8.63, 6.86, 9.54, 8.81, 8.93, 7.67 \)

**Site 2**
1. thickness of weak floor stratum:
   \( H = 1.14 \text{ m} \)
2. moisture content (%):
   \( MC = 8.44, 6.10, 8.31, 8.72, 8.37, 7.58 \)

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Table 2
Geotechnical properties of the overburden strata

<table>
<thead>
<tr>
<th>Type of Rock</th>
<th>Thickness (m)</th>
<th>Modulus of elasticity (MPa)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tan Clay</td>
<td>14.02</td>
<td>3.45E1</td>
<td>0.36</td>
</tr>
<tr>
<td>Sand, Gravel</td>
<td>9.14</td>
<td>1.50E2</td>
<td>0.25</td>
</tr>
<tr>
<td>Gray Clay</td>
<td>23.16</td>
<td>6.96E1</td>
<td>0.30</td>
</tr>
<tr>
<td>Sand, Gravel</td>
<td>2.74</td>
<td>2.00E2</td>
<td>0.25</td>
</tr>
<tr>
<td>Limestone/Shale</td>
<td>6.10</td>
<td>8.62E3</td>
<td>0.15</td>
</tr>
<tr>
<td>Shale/Limestone</td>
<td>16.16</td>
<td>6.62E3</td>
<td>0.20</td>
</tr>
<tr>
<td>Coal</td>
<td>0.51</td>
<td>1.03E3</td>
<td>0.30</td>
</tr>
<tr>
<td>Shale/Limestone</td>
<td>11.89</td>
<td>6.62E3</td>
<td>0.20</td>
</tr>
<tr>
<td>Limestone</td>
<td>1.52</td>
<td>8.62E3</td>
<td>0.15</td>
</tr>
<tr>
<td>Black Shale</td>
<td>0.51</td>
<td>4.56E3</td>
<td>0.20</td>
</tr>
</tbody>
</table>

The average weak floor stratum thickness and corresponding average moisture content used in the analysis are given below.

$H=1.21$ m and $W=8.57\%$

The approximate time-dependent deformability modulus $E_2$

(For the load level equal to 50% of the weak floor strata ultimate bearing capacity) was determined using

Eq.4 (Paia et al., 1960): the equation was developed based on several hundred plate loading tests conducted in about 10 Illinois mines:

$$E_2 = 366.1 \times (M/C)^{0.47} \times (H/1.5)^{0.47}$$

1.33E2 MPa

The time-dependent deformability parameters of the floor strata were estimated based on the surface subsidence data (see Figure 3 and Table 3) monitored at the monument S3 located on the North-South survey line. The standard single Burgers' model was utilized to model the weak floor strata visco-elastic behavior which in mathematical form can be expressed as (Pytel and Chugh 1989):

$$s(t) = [E_2 - E_1] \times \left(1 + \frac{E_1}{E_2} \times \exp \left(-\frac{t}{N_1}\right)\right) + E_2 \times \frac{t}{N_2}$$

where: $s(t)$ is time-dependent subsidence due to settlement of pillars on weak floor, $t$ is time elapsed since mine development, $E_1$, $E_2$, $N_1$, $N_2$ are time-dependent parameters for weak floor strata, and $s(0)$ is time-independent elastic pillar settlement which may be calculated assuming validity of the tributary area

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E₁ = 1.23E2 MPa, E₂ = 1.33E2 MPa
N₁ = 2.74E4 MPa day, N₂ = 6.67E4 MPa day.

Subsidence estimation

Time dependent surface subsidence was investigated based on rheological properties of weak floor strata. All mine workings developed in the total period of time were divided into 31 phases with assigned extraction time (see Figure 2) and into 66 zones of specified percent extraction. The subsidence analysis was conducted for periods of 101 and 696 days after subsidence monitoring equipment was installed.

The surface subsidence contours over T=101 days and T=696 days are shown in Figures 4 and 5. The calculated and monitored subsidence along the survey lines N-S and E-W, located over the panels (see Figure 2) are presented in Figures 6 and 7.

The results of the calculation are in agreement with the corresponding data monitored in the area, particularly at the points located over the considered panels. The differences are less than 5 per cent for T=696 days and less than 14 per cent for T=101 days. The relatively large differences near the barrier pillars are probably due to uncertain data concerning the extraction schedule in those areas.

Fig. 4 - Vertical subsidence [cm] contours over the mined-out areas of Mine 1 after 101 days.

Fig. 5 - Vertical subsidence [cm] contours over the mined-out areas of Mine 1 after 696 days.

Fig. 6 - Comparison of the predicted and observed subsidence profiles over Mine 1 (N-S direction).

Fig. 7 - Comparison of the predicted and observed subsidence profiles over Mine 1 (E-W direction).

MINE II

Mine description

The mining layout consisting of two panels was extracted using the conventional room-and-pillar mine development followed by secondary mining of pillar in a slab mode. Layout of mine workings and subdivision of the mined-out areas into phases and zones are shown in Figure 8. The first indication of any floor instability problems in the extracted panel zones 4-
38) became evident shortly after completing secondary mining in zones 10-13 (December, 1986). The observed floor instability was mostly confined to the secondary mined-out areas and did not threaten other mine workings. However, approximately six months after the panel pillars were secondary mined (June, 1986), the floor movements extended to the mouth of the panel (zone 31) where convergence stations were installed on January 17, 1989, to monitor these movements. Until August 30, 1989 a total convergence of approximately 8.08 cm was recorded. This information was the only subsidence/settlement data available from this region, and was the basis for the developed three-dimensional model validation. The average mining geometry and available geotechnical parameters used in the analysis are given below.

1. overburden thickness: \( H_0 = 264.5 \text{ m} \)
2. overburden pressure before mining: \( q_0 = 3.58 \text{ MPa} \)
3. Poisson’s ratio for weak floor strata: \( v = 0.25 \)
4. average mining height: \( H_4 = 2.18 \text{ m} \)
5. modulus of elasticity for coal: \( E = 1.21 \text{ 133 MPa} \)
6. extraction ratio: \( e = 0.329 - 0.652 \)

7. opening widths: \( W = 5.5 \text{ m} \) (development) or \( 13.5 \text{ m} \) (secondary mining)
8. square pillar size: \( B = 25 \text{ m} \) (development) or \( 18 \text{ m} \) (secondary mining)

**Overburden stiffness characterization**

The overburden cross section above the coal seam consists of 32 layers of different types of rocks, from glacial deposits, clay and shale to sandstone and limestone. The calculated overburden flexural stiffness \( D \) is given below.

\( D_1 = 1.586 \text{ MNm}^{-3} \) (fully bonded layers) and \( D_2 = 6.366 \text{ MNm}^{-3} \) (perfectly smooth interfaces) and the latter value was utilized in the analysis.

**Weak floor strata characterization**

The data concerning weak floor stratum thickness \( H \) and its moisture content \( MC \) is given below.

**Site 1**
- thickness of weak floor stratum: \( H = 1.32 \text{ m} \)
- moisture content \( MC \): 13.74 5.18 2.09 7.09 7.07 7.27 7.42

**Site 2**
- thickness of weak floor stratum: \( H = 1.72 \text{ m} \)
- moisture content \( MC \): 7.07 11.49 16.39 4.15 3.69 7.56 10.75

Therefore, the average weak floor stratum thickness and corresponding average deformability modulus \( E_2 \) calculated from Eq. 4 are as follows:

\( H = 1.52 \text{ m}, \ E_2 = 1.0022 \text{ MPa} \)

The value of \( E_2 \), as previously identified, governs instantaneous elastic displacements due to overburden pressure. The remaining floor time-dependent parameters were estimated based on the average data from the Illinois Coal Basin. Since the subsidence prediction was done for the time period long after mine workings were completed, the contribution of the Kelvin-Voigt unit and its parameters \((R, N)\) on subsidence was assumed to be insignificant. Therefore, the Burgers model was reduced to the single Maxwell unit with the following parameters:

\( E_2 = 1.0022 \text{ MPa} \) and \( N_a = 2.654 \text{ MPa} \)

**Pillar settlement at the convergence station**

All mine workings developed in the total period of time \( T = 1403 \text{ days} \) were divided into 12 phases with
assigned extraction time (see Figure 6) and into 28 zones of specified percentage extraction. The total subsidence analyses was conducted for a period of 1463 days after mine workings were initiated. Additional incremental surface subsidence developed between the 1179th and 1463rd day was also analyzed to relate it to corresponding in-mine monitored roof-floor convergence.

The utilized three-dimensional analysis model proved that settlement of pillars located in the vicinity of convergence points was equal to 1.94 cm. The corresponding pillar displacement may be also calculated using Eq 5 as follows:

\[ Y = C/\left(1-n\right) = 5.08/\left(1+1.71\right) = 1.94 \text{ cm} \]

where coefficient \( c \) was taken from Figure 1 as suitable for \( H/B=1.52/25=0.07 \) and for \( B/W=25/5.5=4.55 \). The agreement between results obtained from convergence data and those calculated using three-dimensional time-dependent model (Figure 5) is extremely good.

**Fig. 9** - Total and incremental surface subsidence profiles along cross section E-W over Mine II.

**CONCLUSIONS**

The validity of the developed SUU Subsidence Prediction Model (SUU PANEL.3D) was confirmed at two central Illinois mines. Time-dependent deformability parameters of weak floor strata calculated from the observed surface subsidence data are reasonable and may be used for predicting subsidence in room-and-pillar mine workings using the SUU PANEL.3D model, particularly for a longer period of time. The model requires accurate time sequence of extraction of mine workings and mine geometry to predict subsidence.

**REFERENCES**


11th International Conference on Ground Control in Mining. The University of Wollongong, N.S.W., July 1992.