CHAPTER THREE

CLOSURE AND STRENGTH OF LINING REQUIRED IN DRIVAGES
3.1 Introduction

Calculations as to the required strength of drivage linings in unconsolidated ground near the surface are based largely on the work of Terzaghi [21]. He considered the stability of a column of ground above the excavation as follows (see Figure 5): the zone Z liable to slip is kept in place by the internal friction F acting at the sides of the zone, augmented by the upward resistance of the supports P. Horizontal pressure from the supports is also required in order to maintain the stability of the sides. Below a relatively shallow depth, the formula derived is independent of the depth itself. It is usual to assume equal virgin horizontal and vertical pressures, and to ignore any cohesion present in the ground. On these premises, the required vertical pressure to be supplied by the support is given by

\[ \sigma_v = \frac{\gamma}{2 \tan \phi} \left[ w + 2h \tan (45^\circ - \frac{1}{2}\phi) \right] \]  \hspace{1cm} \ldots(3.1.1)

and the required horizontal pressure by

\[ P_h = 0.3 \left( \frac{1}{3} h \gamma + \sigma_v \right) \]  \hspace{1cm} \ldots(3.1.2)

where \( h \) is the height and \( w \) the width of the excavation, \( \phi \) is the angle of internal friction of the ground and \( \gamma \) its density. For typical values of \( h, w, \phi \) and \( \gamma \), the value of \( P_h \) is approximately half that of \( \sigma_v \).

When considering drivages in rock, the vertical resistance required from the lining was expressed by Terzaghi [22] as the apparent height of the ground requiring support, i.e.

\[ H = \frac{P_v}{\gamma} \]

If
FIGURE 5. ZONE OF ARCHING ACCORDING TO TERZAGHI
an angle of internal friction \( \phi = 37^\circ \) is assumed, then, from Equation 3.1.1, \( H = 0.67 (h + w) \). To take into account the natures of different types of rock formation Terzaghi varied the multiplier of \( (h + w) \) from 0 to 1.1. In the case of 'squeezing rock at great depth', it was increased further to 4.5. (NB. Terzaghi considered 240 m to be 'great depth' [23].) If the excavation is of the order of 10 m x 10 m, then \( (h + w) = 20 \) m, and a multiplier of 4.5 will postulate a constant value of required support resistance below a depth of 90 m.

In the British coal mining industry, drivages are carried out to depths of 1100 m, with deeper drivages being considered. Rocks surrounding drivages are frequently weak, and it is recognised that 'yield zones' develop around the excavations and that these cause closure of the openings. The degree of lining strength required is known to depend on the strength of the rock, on the depth of cover and on the amount of closure allowed before inserting the lining. Although the theories of Terzaghi have proved their worth in the design of linings for shallow depth (less than 100 m), they are obviously inapplicable at greater depths. To use them so is to use them out of context. In this section the author intends to consider the factors pertaining to the support of drivages in soft rock at depth, and to develop formulae which will give estimates of lining strengths required in given situations.

3.2 Assumptions

In order to obtain convenient, manageable equations and to simplify the calculations, certain assumptions must be made. Although these assumptions may not apply in practice, the formulae produced can be used as a guide to deduce the conditions likely to occur. This will be further discussed later in the Chapter.
The basic assumptions are:

(a) the drivage has a circular cross-section,
(b) it is surrounded by homogeneous isotropic rock,
(c) plane strain conditions exist, and
(d) the virgin horizontal and vertical stresses are equal.

If purely elastic conditions prevail right to the boundary of a circular excavation, the tangential stress at the boundary will be twice the virgin hydrostatic stress field [24]. At depth this stress will generally exceed the strength of the average Coal Measure rock, and a yield zone adjacent to the boundary will develop. Certain assumptions can be made as to the failure criteria both in the yield zone and in the elastic zone beyond.

Although the relationship between stress at failure $\sigma_1$ and the confining pressure $\sigma_3$ is frequently curved, an approximate linear relationship of the form

$$\sigma_1 = \sigma_0 + k \sigma_3$$  

...(3.2.1)

can be assumed for unbroken rock, where $\sigma_0$ is the unconfined compressive strength and $k$ is a constant for the particular rock, here referred to as the 'triaxial stress factor'.

A similar criterion is also assumed in the failed rock within the yield zone, ie

$$\sigma'_1 = \sigma'_0 + k \sigma'_3$$  

...(3.2.2)

In this case $\sigma'_1$ is the stress required to cause movement in the broken material when confined by a pressure $\sigma'_3$, and $\sigma'_0$ is the corresponding stress at zero confinement.
Throughout the yield zone the rock will be in a state of failure and Equation 3.2.2 not only represents the failure criterion, but also the relationship between the major and minor principal stresses $\sigma_1$ and $\sigma_3$ at any point. Diminution of the confining pressure $\sigma_3$ will cause movement within the yield zone. This movement will continue until stable conditions are again established. In the unbroken zone beyond the yield zone, except at the boundary, the rock will be below its failure limit and the laws of elasticity will apply.

Equations 3.2.1 and 3.2.2 are identical to the Rankine equation for active earth pressure used in Soil Mechanics. The triaxial stress factor $k$ is a function of the angle of internal friction $\phi$, and, as shown in Section 2.2, can be expressed by

$$ k = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left( 45^\circ + \frac{\phi}{2} \right), \text{ or} $$

$$ \frac{1}{k} = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right), \text{ or} $$

$$ \tan \phi = \frac{k - 1}{2\sqrt{k}}. $$

These relationships will be used in Section 3.7.

3.3 Stress in the Yield and Elastic Zones

Stress in a yielding zone, where the failure condition is given by $\sigma' = \sigma_0' + k\sigma_3'$, has been analysed by a number of investigators in Soil Mechanics, notably by Westergaard [25], Terzaghi [26] and Nadai [27]. Hobbs [14] introduced a curved relationship between failure and confining pressure, but his formulae are of such complexity as to make their application difficult.
At the request of the author, Airey [28] re-analysed the stress conditions in the yield and elastic zones, assuming the failure criterion given by Equations 3.2.1 and 3.2.2. The resulting stress relationships can be written as follows:

(a) **Within the yield zone**

 radial stress \( \sigma_r = (p + p') \left( \frac{r}{r_0} \right)^{k-1} - p' \) \( \ldots (3.3.1) \)

tangential stress \( \sigma_\theta = k(p + p') \left( \frac{r}{r_0} \right)^{k-1} - p' \) \( \ldots (3.3.2) \)

where \( r_0 \) is the radius of the opening, \( p \) the restraint on its boundary and \( p' = \frac{\sigma_0}{k - 1} \).

(b) **At the yield/elastic boundary**

 \( \frac{\sigma_r}{r} = \frac{2q - \sigma}{k + 1} \) \( \ldots (3.3.3) \)

 \( \sigma_\theta = \frac{k(2q - \sigma)}{k + 1} + \sigma_0 \) (on yield side of boundary) \( \ldots (3.3.4) \)

 \( \sigma_\theta = \frac{k(2q - \sigma_0)}{k + 1} + \sigma_0 \) (on elastic side of boundary) \( \ldots (3.3.5) \)

where \( q \) is the hydrostatic stress field remote from the opening.

(c) **Within the elastic zone**

 \( \sigma_r = q - A \left( \frac{r}{r_0} \right)^2 \) \( \ldots (3.3.6) \)

 \( \sigma_\theta = q + A \left( \frac{r}{r_0} \right)^2 \) \( \ldots (3.3.7) \)

where \( A = \frac{(k - 1)q + \sigma_0}{k + 1} \cdot \frac{2q - \sigma_0 + p' (k + 1)}{(p + p') (k + 1)} \) \( \ldots (3.3.8) \)
By combining Equations 3.3.1 and 3.3.3, the radius $\bar{r}$ of the yield/elastic boundary can be shown to be

$$
\bar{r} = r \left( \frac{2q - c + p'}{(p + p') (k + 1)} \right)^{\frac{1}{k-1}} \quad \ldots (3.3.9)
$$

The general form of the stress distribution is shown in Figure 6.

At this juncture it is convenient to calculate the closure of the yield/elastic boundary brought about by the excavation. From any standard text-book on the theory of elasticity it can be deduced that within the elastic zone under conditions of plane strain,

$$
tangential \ strain = \frac{\Delta u}{r} = \frac{1}{E} \left( \frac{\sigma_\theta - \nu \sigma_\rho}{r} - \nu \sigma_\rho \right) \quad \ldots (3.3.10)
$$

where $u$ is the radial displacement at radius $r$, $E$ is the modulus of elasticity and $\nu$ is Poisson's ratio.

The radial movement $\bar{u}$ of the elastic zone boundary $\bar{r}$, as a result of the excavation, will be that produced by the change in the stress field from the original value $q$. From Equations 3.3.6 and 3.3.7 we have, at the elastic boundary,

- change in radial stress $= -A \left( \frac{r^2}{\rho} \right)$
- change in tangential stress $= +A \left( \frac{r^2}{\rho} \right)$

Therefore, from Equation 3.3.10, we have

$$
\left\{ \frac{\Delta u}{r} = \frac{1}{E} \left[ A \left( \frac{r_0}{\rho} \right)^2 + \nu \cdot A \left( \frac{r_0}{\rho} \right)^2 \right] \right\}
$$
Figure 6. Stresses around a circular roadway surrounded by a yield zone.
\[ \vec{u} = A \mathbf{r}_0 \cdot \left( \frac{\mathbf{r}}{r} \right) \cdot \frac{1 + \nu}{E} \quad \ldots (3.3.11) \]

where \( A \) is defined by Equation 3.3.8.

### 3.4 Expansion in the Yield Zone

Movement of individual pieces of broken rock relative to one another within the yield zone will produce voids, the total void volume probably increasing as the movement increases. Little is known about the specific relationship between this expansion and the movement, but some indication can be logically deduced.

Consider an element of unit thickness, as shown in Figure 7. Let a fractional contraction \( s \) produce a fractional extension \( t \) normal to \( s \). If the bulking factor \( B \) is defined as the ratio of the rock volume after fracture to the volume before, and \( v \) is the fractional increase in void volume, ie the increase in void volume divided by the original void volume, then

\[ B = 1 + v \]

The value of \( v \) will probably depend on the amount of distortion \( s \). For a given pressure, assume a linear dependence,

\[ v = F_1 s \quad \ldots (3.4.1) \]

where \( F_1 \) is a function of the pressure. In the yield zone the ratio of the principal stresses \( \sigma_1 \) and \( \sigma_3 \) is approximately constant and equal to \( k \). Therefore, \( F_1 \) can be chosen as a function of either stress without altering the form of the function. For convenience, in its experimental determination, \( F_1 \) has been chosen as a function of \( \sigma_3 \).
FIGURE 7. DISTORTION OF ELEMENT IN YIELD ZONE
If we again return to Figure 7, then volume after distortion =

B times volume before distortion

\[ \text{Volume}_1 (1 - s) \cdot \text{Volume}_2 (1 + t) = (1 + \text{ase}^{\frac{\sigma}{b}} - 1) \cdot \text{Volume}_1 \cdot \text{Volume}_2 \]

\[ t = \frac{s}{1-s} (1 + \text{ase}^{-\frac{\sigma}{b}}) \]  \hspace{1cm} ...(3.4.4)

Figure 7, however, represents only one element. Across the

yield zone there will be a number of elements in series, and the
total movement \( u \) between any two points will be \( u = \text{Ax} \cdot \delta x \)
(represented in Figure 8a). If \( t \) is a function of \( x \) as
represented in Figure 8b, then \( \delta u = t \cdot \delta x \) and \( \frac{du}{dx} = t \). In a
circular roadway, increase of \( r \) will be in the opposite sense to
increase in \( x \) and \( u \), and \( dr = -dx \)

\[ \text{dr} \]

\[ \frac{du}{dr} = t \]  \hspace{1cm} ...(3.4.5)

Consider an element in the yield zone, (a) before yield
occurs (b) after yield (as shown in Figure 9)

\[ s = \frac{\text{original } \text{Volume}_1 - \text{final } \text{Volume}_2}{\text{original } \text{Volume}_1} \]

\[ s = \frac{\delta r - \delta (r - u)}{\delta r} = \frac{u}{r} \]  \hspace{1cm} ...(3.4.6)

Combining Equations 3.4.4, 3.4.5 and 3.4.6 gives a
differential equation of the form

\[ \frac{du}{dr} = \frac{\sigma}{r-u} (1 + \text{ase}^{\frac{\sigma}{b}} - \frac{1}{r}) \]  \hspace{1cm} ...(3.4.7)

where (from Equation 3.3.1) \( \frac{\sigma}{r} = \sigma = (p + p') \frac{r}{r_0} \]

\[ - 24 - \]
FIGURE 8. EXPANSION OF A SERIES OF ELEMENTS IN THE YIELD ZONE
FIGURE 9. MOVEMENT AND RESULTANT CONTRACTION IN A CIRCULAR YIELD ZONE
Granulated rock fills approximately 1.5 times its volume in the unbroken state, i.e., the fractural void volume formed is about 0.5 of the original volume. If the broken rock is now subjected to increasing pressure, it will gradually compress back to its original volume, the rate of compression being approximately proportional to the void volume remaining. (The results of compressing a granulated mudstone are shown in Figure 10.) This suggests an association between void volume \( v \) and the pressure \( \sigma_3 \) at constant \( s \) of the form

\[
v = \frac{\sigma_3}{2} \quad \text{...(3.4.2)}
\]

Combining Equations 3.4.1 and 3.4.2 gives

\[
v = a \omega_0 \frac{\sigma_3}{b} \quad \text{...(3.4.3)}
\]

where \( a \) and \( b \) are constants depending on the type of rock.

At the request of the author, the University of Newcastle upon Tyne is undertaking a study of the expansion of rock specimens following failure. The results of a preliminary investigation [28] in Portland Limestone are shown in Figure 11a. Difficulty was experienced in measuring low values of \( v \), but, within the limits of the experiment, \( v \) appears proportional to \( s \) for a given value of \( \sigma_3 \). Figure 11b shows \( \log_{10} v \) plotted against \( \sigma_3 \) for \( s = 0.1 \), from which it can be deduced that

\[
v = 0.474 s \exp \left( \frac{-\sigma_3}{12.1} \right) \quad \text{with} \quad \sigma_3 \text{ in MPa}
\]

which has the same form as Equation 3.4.3. In the limit if \( \sigma_3 = 0 \), then \( v \approx 0.5s \).
FIGURE 10. COMPRESSION OF GRANULATED MUDSTONE GRAIN SIZE 5 TO 10 mm
(DEDUCED FROM RESULTS OBTAINED BY PRICE AND MAY [36])
FIGURE 11. POST-FAILURE EXPANSION OF PORTLAND LIMESTONE
The solving of this differential equation, however, is difficult, and although solution by computer program for specific values of the variables is possible, simplification was thought desirable.

The complexity arises from the term \(ae^{-b}\) and therefore a simplifying approximation for this expression was considered.

Let \(ae^{-\frac{\sigma_2}{b}} = \varepsilon\). Hence from Equation 3.4.3, the fractional increase in volume \(v = \varepsilon s\). As the stress in the yield zone decreases from the yield/elastic boundary to the excavation boundary, there will be at least elastic expansion, and as the distortion \(s\) is positive and real, the lower limit of \(\varepsilon\) must be greater than zero. As the bulking factor of granulated rock is approximately 1.5, the maximum value of \(\varepsilon s\) will be approximately 0.5. This will probably occur well before \(s = 1\) (i.e. before the element is squashed completely flat). The upper limit of \(\varepsilon\), therefore, must be less than 0.5.

Hence \(0 < \varepsilon < 0.5\)

Let it be assumed that the value of \(\varepsilon\) can be approximated by a constant lying between 0 and 0.5. The constant will be referred to as the expansion factor, and may vary according to the type of rock.

With this approximation, Equation 3.4.7 can now be re-written as

\[
-\frac{du}{dr} = \frac{u}{r-u} (1 + \varepsilon)
\]

which has the solution

\[
u \left(\frac{r(2+\varepsilon)}{r-u} - u\right)^{1+\varepsilon} = \tilde{u} \left(\frac{r}{r(2+\varepsilon)} - u\right)^{1+\varepsilon} \quad \ldots (3.4.8)
\]

where \(\tilde{u}\) is the movement of the yield/elastic boundary at radius \(\tilde{r}\).
Although this equation can be solved by graphical or by computer methods, a rigorous solution for \( u \) is not possible without further simplifying approximations.

(a) Consider \( \epsilon \) very small when compared to 1. Equation 3.4.8 then reduces to

\[
(2\tau - u) = \tilde{u} (2\tau - \tilde{u})
\]

ie

\[
u = r - (r^2 - 2\tilde{u}r + \tilde{u}^2)^{1/2}
\]

...(3.4.9)

This relationship can be proved independently by reference to Figure 12.

Initial volume of yield zone = \( \pi \tau^2 - \pi \tilde{r}^2 \)

Final volume of yield zone = \( \pi (r - \tilde{u})^2 - \pi (r - u)^2 \)

By equating these volumes and solving for \( u \), Equation 3.4.9 can be reproduced.

(b) Roadway linings are normally designed to keep the movement \( u \) small when compared to the diameter of the roadway. In this case \( u \) will be small compared to \( 2r \), and Equation 3.4.8 may be approximately re-written as

\[
u \left( \frac{r}{r(2 + \epsilon)} \right)^{1+\epsilon} = \tilde{u} \left( \frac{r}{r(2 + \epsilon)} \right)^{1+\epsilon}
\]

from which

\[
u = \tilde{u} \left( \frac{r}{r(2 + \epsilon)} \right)^{1+\epsilon}
\]

...(3.4.10)

This will be pursued further.
The value of $\bar{u}$ is the movement of the yield/elastic boundary, inside which we assume the above considerations to apply. Should subsequent research show that there is an immediate expansion at yield followed by a lesser expansion as distortion occurs, then the immediate expansion can be added to the elastic movement, and $\varepsilon$ will be the smaller value associated with the distortion.

Until it is known otherwise, let $\bar{u}$ be the elastic movement only, as given in Equation 3.3.11. Combining Equation 3.3.11 with Equation 3.4.10, the displacement of roadway boundary $r_o$ is given as

$$u = r_o \cdot \frac{1+\nu}{E} \cdot A \left( \frac{r}{r_o} \right)^n$$

Further substitution for $A$ from Equation 3.3.8 and $\frac{r}{r_o}$ from Equation 3.3.9 gives

$$u = r_o \cdot \frac{1+\nu}{E} \cdot \left( \frac{k-1}{k+1} \right) \cdot \frac{(k+1) q \pm c}{(k+1)}$$

$$= \left( \frac{2q - c + p' (k+1)}{(p + p') (k+1)} \right) \frac{2\varepsilon}{\omega q}$$

... \text{(3.4.11)}

3.5 Consideration of $\frac{1+\nu}{E}$, $c_o$, $p'$ and $\varepsilon$

(a) Elasticity Factor, $\frac{1+\nu}{E}$

In many of the softer rocks the modulus of elasticity $E$ and the Poisson's ratio $\nu$ are difficult to obtain. However, an approximation of the expression $\frac{1+\nu}{E}$ in terms of the compressive strength is possible.
In Figure 13, the values of the moduli of elasticity from 176 rock samples tested at the National Coal Board’s Mining Research and Development Establishment are shown, plotted against the laboratory-determined, unconfined compressive strength. The range of rocks covered included sandstone, siltstone, mudstone, silty mudstone, marl and coal with shale, but not coal itself. An approximate linear proportionality occurs. Therefore, as a rough guide, it can be taken that \( E = \sigma \times 0.31 \times 10^3 \).

No relationship could be established between \( \nu \) and \( \sigma \), a scatter of results occurring between \( \nu = 0.1 \) and \( \nu = 0.4 \), but \( \nu = 0.25 \) was a good representative value. As \( \nu \) is added to 1 in the expression in question, its value is not as critical as that of \( E \).

Hence, if actual measured values of \( \nu \) and \( E \) are not available, then as an approximation for strata other than coal it can be accepted that

\[
\frac{1 + \nu}{E} = \frac{1.26}{\sigma \times 0.31 \times 10^3} = \frac{4}{\sigma} \times 10^{-3}
\]

In the case of uncontaminated coal, the modulus of elasticity varies little from 3.5 GPa and the Poisson's ratio little from 0.3, but the strength may vary from 1 MPa to 40 MPa. Hence, if dealing with a drivage completely surrounded by coal, and if actual values are not known, it should be assumed that as an approximation \( \frac{1 + \nu}{E} = 0.37 \times 10^{-3} \text{ MPa}^{-1} \). This figure is substantially higher than the value for average strata, and may explain the higher than average closure of roadways in the Warwickshire Thick coal seam.
FIGURE 13. RELATIONSHIP BETWEEN ELASTICITY MODULUS AND COMPRESSION STRENGTH (STRATA OTHER THAN COAL)
(b) In situ Unconfined Compressive Strength, $\sigma_u$

The parameter $\sigma_u$ is the unconfined compressive strength of
the rock in situ in the elastic zone, complete with the
attendant joints, cleats, bedding planes and other weaknesses.
It is not the unconfined compressive strength as determined in
the laboratory. For laboratory determination, unbroken specimens
between joints or other major fractures are selected for core
preparation. Cores which break in preparation are rejected.
The smaller the sample, the less the chance of finding a plane
of weakness. Hence the laboratory test is severely biased 'on
the strong side'.

The results outlined in published work on the strength of
coal (see Section 2.1) prompted the conclusion that with a closely-
cleated rock, the strength varies approximately as the inverse
of the cube root of the specimen dimension. Allowing for
differences between laboratory specimen size and roadway size
for closely-cleated rock, the laboratory strength should be
divided by 5 in order to obtain $\sigma_u$. In a massive rock with
widely-spaced joints, the dividing factor will probably remain
at unity until the specimen size is greater than the joint spacing.
On the other hand, in a highly-faulted area the dividing factor
could well exceed 5.

Further investigation into the differences between the in-
situ and laboratory strength is required, including any relation-
ship with the Rock Quality Designation (RQD) [30] factor commonly
used in geotechnical engineering, to define the broken
state of a core. For the time being, however, it is suggested
that the unconfined laboratory strength $\sigma$ be divided by a
factor \( f \), where

\[
f = \begin{cases} 
1 & \text{for strong massive unjointed rock (including concrete)}, \\
2 & \text{for widely-spaced joints or bedding planes in strong rock}, \\
3 & \text{for more jointed, but still massive rocks}, \\
4 & \text{for well-jointed and weaker rocks}, \\
5 & \text{for unstable seat earths and closely-cleated rock such as coal}, \\
6 \text{ and } 7 & \text{for weak rock in the neighbourhood of fault zones.}
\end{cases}
\]

This concept is not vastly different from that adopted by Terzaghi [22] and others. The weaker rocks, regardless of jointing, are given the higher values since here the bias introduced by sample selection and specimen preparation is particularly high.

(c) Augmentation to the Lining Strength, \( p' \)

The parameter \( p' \) was defined in Section 3.3 as

\[
p' = \frac{c'}{k-1}
\]

where \( c' \) represents a quantity corresponding to the unconfined compressive strength of broken material in Equation 3.4.2. A value of \( c' \) may result from the broken pieces locking together, or from the approximation by a straight line of a curved relationship between \( q_1' \) and \( q_2' \) producing a positive intercept on the \( q_1' \) axis.

If the results published by Hobbs [14] for the properties of broken rocks are represented by straight lines, then the values of \( c' \) are of the order of 5 to 10 MPa. However, these results are obtained by carefully replacing the broken fragments in their original positions, as the result of which there is a certain 'keying in', especially where values of confining pressure other
than that required to fracture the specimen are used. If \( k \) varies with \( \phi \), so too will the angle of shear, and the fragments will attempt to move in a plane at variance to the original shear breaks. On the other hand, Oram [17] in his research, used granular materials. These give much lower values, ranging from 0 for anthracite to 0.6 MPa for Pennant Sandstone, but averaging approximately 0.2 MPa for the softer rocks and 0.3 MPa for the harder. The corresponding values of \( k \) are 3 and 4, which suggests a value of \( p' \) of 0.1 MPa. Until there is evidence to the contrary, it is proposed to err on the side of safety and to adopt this latter value.

From Section 3.5(b) it is obvious that the value of \( \phi \) will have a probable large experimental error compared to the value of \( p' (k + 1) \). Hence, as an approximation, this latter term can be omitted from the numerator of the final part of Equation 3.4.11. However, \( p' \) is comparable to \( p \), and the term \( (p + p') \) must remain in the denominator. In essence, it represents an augmentation of the strength of the lining, hence the symbol attributed to it.

(d) Expansion Factor, \( \varepsilon \)

In Section 3.4 it had to be assumed that \( \varepsilon \) was a constant, and it was shown that \( 0 < \varepsilon < 0.5 \). In Equation 3.4.11 \( \varepsilon \) appears in the form \( 2 + \varepsilon \); therefore the equation is not critical for small variations in \( \varepsilon \). It is proposed, unless and until it is shown to be otherwise, that \( \varepsilon \) be assigned a value of 0.2. The line corresponding to this value is shown in Figure 9a.
3.6 Final Form of the Equation

By incorporating the suggestions made in Section 3.5, the final form of the equation linking the closure to the lining strength and to the rock properties can be rewritten as

$$ c = d \cdot \frac{1 + \nu}{E} \cdot \left( \frac{(k+1) q + \frac{q}{f}}{(k+1)} \right) \cdot \left( \frac{2q - \frac{q}{f}}{(p + p') (k + 1)} \right) \cdot \frac{\frac{2q}{f}}{k - 1} \ldots (3.6.1) $$

$$ w = \frac{d}{c} \cdot \left( \frac{(k+1) q + \frac{q}{f}}{(k+1)} \right) \cdot \left( \frac{2q - \frac{q}{f}}{(p + 0.1) (k+1)} \right) \times 10^{-3} \ldots (3.6.2) $$

where $c =$ the diametric closure

$d =$ the driven diameter,

$\nu =$ Poisson's ratio,

$E =$ modulus of elasticity,

$k =$ triaxial stress factor,

$q =$ cover load,

$\sigma =$ the laboratory-determined, unconfined compressive strength of rock,

$p =$ the lining resistance,

$f =$ a factor relating laboratory strength to in-situ strength,

taking into account the degree of jointing or fracturing,

$p' =$ an augmentation of the lining resistance brought about by the 'cohesion' of the yielded rock,

$e =$ the expansion factor.

In using Equation 3.6.2, the units must be in metres and MPa.

The parameters $d$ to $p$ can be specified or measured; $f$ can be approximated if one knows the general conditions of the ground through which the tunnel has to be driven. Approximations are inserted in Equation 3.6.2 for $\frac{1 + \nu}{E}$, $p'$ and $e$. 
3.7 Minimum Vertical Support Required

In developing the formula, the weight of ground local to the excavation was ignored, because it was small when compared to the total weight of the cover load. However, a certain minimum upward thrust must be provided by the lining in order to secure the fractured material in the yield zone above.

Two methods of calculating the minimum thrust will be considered; one was developed by Terzaghi for granular material, the other by Airey for broken stratified material.

(a) Granular Material

If the material in the yield zone can be considered to be granular, the laws of Soil Mechanics will apply. According to Terzaghi [21], the maximum downward pressure to be resisted (regardless of height of yield zone) will be

\[ p_v = \frac{\gamma B_1}{k \tan \phi} \]

In the case of that portion of yield zone forming the roof, the individual terms will have the values

\[ B_1 = \frac{d}{2} + d \tan (45^\circ - \frac{\phi}{2}) \]

\[ = d \left( \frac{1}{2} + \frac{1}{\sqrt{k}} \right) \] (see Section 2.2)

\[ K = \frac{\sigma_m}{\sigma_v} = \frac{\sigma_m}{\sigma_p} \]

\[ \tan \phi = \frac{k - 1}{2 \sqrt{k}} \] (see Section 2.2)

\[ \therefore \quad \frac{p_v}{d} = \frac{\gamma (\sqrt{k} + 2)}{k (k - 1)} \] \[ \ldots \text{(3.7.1)} \]
(b) Broken Stratified Material

Airey [31] made a mathematical study of the amount of broken material released when highly stratified ground is over-stressed and a yield zone surrounds the whole roadway. He found that the strata buckled (as is shown in Figure 14), the angle of break to the vertical being the same as the angle of internal friction $\phi$.

\[
\text{Area of broken ground} = \frac{d^2}{4 \tan\phi}
\]

\[
\therefore \text{weight of broken ground} = \frac{d^2 \gamma}{4 \tan\phi} \text{ per unit length of run.}
\]

In terms of the average pressure, this gives

\[
\frac{P_x}{d} = \frac{\gamma}{4 \tan\phi} = \frac{\gamma \sqrt{k}}{2(k-1)} \quad \cdots (3.7.2)
\]

Despite the widely different hypotheses, Equations 3.7.1 and 3.7.2 give similar results for $k = 3$ to $k = 5$, and an identical result if $k = 4$, viz,

\[
\frac{P_x}{d} = \frac{\gamma (\sqrt{4} + 2)}{4(4 - 1)} \quad \text{or} \quad \frac{\gamma \sqrt{4}}{2(4 - 1)} = \frac{\gamma}{3}
\]

As the stratification of the Coal Measures is likely to be pronounced, it is proposed that Equation 3.7.2 be adopted.

Figure 15 shows, according to Equation 3.7.2, the minimum vertical support resistance required to hold the broken ground above a roadway over the range of the value of $k$ usually encountered. It is known from experience that 20 tonnes per arch leg at 1 m spacing is sufficient to support a 5 m wide roadway. This corresponds to a resistance of 0.08 MPa, and is in line with the value obtained from the graph.
FIGURE 14. POSSIBLE AREA OF BROKEN GROUND ABOVE ROADWAY IN STRATIFIED ROCK (ACCORDING TO AIREY) [31]
FIGURE 15. MINIMUM REQUIRED VERTICAL SUPPORT RESISTANCE IN A ROADWAY DRIVAGE
3.8 Use of the Equation in Practice

The Equation 3.6.1 is based on the assumption that a roadway of circular cross-section is driven in a homogeneous isotropic rock. In practice, this may not apply. The formula, however, can still be used as a means of indicating the conditions which are likely to occur. Drivages which are not circular in shape will produce distortions in the immediate stress field, especially at the corners of the opening. However, provided the width of the opening is similar to its height, the yield/elastic boundary will still be roughly circular in shape. Hence the formula will give approximation for square and arch-shaped roadways.

In roadways supported by arches only, no physical support is given to the floor apart from the 'cohesive' effect $p'$. The yield zone will still form, but all the resultant closure will probably occur as floor lift. Even this can offer a restraint to further movement, a pressure of 0.025 MPa per metre of heaved material, which is far from insignificant, being generated.

Stratification will also have an influence. Yield will occur preferentially in the weaker strata, causing distortion in the pattern of closure. The lining, or its method of installation, may have to be modified to take into account such local effects. Buckling of the roof and floor strata may also occur because of horizontal ground stresses.

The formula gives the total ground closure to be expected with a given strength of lining — linings with a built-in facility to yield need not be as strong as rigid linings. The closure to be allowed for or resisted by the lining will depend on how far back from the end of the drivage it is to be installed.
and the fit of the lining in the excavation. A lining installed
tight to the rock and very close to the road-end will reduce
closure, but a very strong lining will be required. Unless there
is a significant time-creep effect (as distinct from increasing
distance from the road-end), the bulk of unresisted closure will
be complete at about 5 or 6 diameters back from the road-end.
Thus a lining installed in the region of two diameters back
will only have to resist approximately half the calculated closure.
A lining installed following the repair of an old roadway need
resist only very little closure as the yield zone will already
have been established. Consequently the lining could be much
reduced in strength.

3.9 Application to Specific Cases

A search of past records did not reveal sites where all the
necessary data had been recorded, a fact which presents
difficulties when it comes to verifying the equation in practice.
In particular, the triaxial stress factor k (or its equivalent,
the angle of internal friction $\phi$) had not been determined.
However, two examples given below show that the equation supplies
the right order of movement observed. A third example shows the
application of the equation as a means of assessing the conditions
likely to be met, say, in the Barnsley Seam at Selby.

(a) Lining Failure at Abernant [6]

At Abernant Colliery in South Wales two insets were formed
at a depth of 720 m in highly faulted strata, composed mainly of
distorted weak mudstones. The initial lining was reinforced
concrete about 1 m thick, but this rapidly broke up. It was
replaced by a second lining strengthened with ring beams of
double reinforced concrete about 2 m thick. These also failed,
and success was not achieved until the roadways had been driven
beyond the area to be lined and the strata allowed to stabilise,
many metres of flowed strata being removed in the interim period.

Typical values for such strata:

triaxial stress factor \( k = 3 \)
laboratory compressive strength \( c = 10 \text{ MPa} \)
in-situ factor (fault zone) \( f = 7 \)

Other relevant parameters:

resistance of steel arches set while strata were
settling \( p = 0.05 \text{ MPa} \)
cover load \( q = 0.025 \times H = 18 \text{ MPa} \)
driven diameter of inset \( d = 9 \text{ m} \)

Insertion of these values in Equation 3.6.2 gives

\[
c = d \frac{4}{c} \left( \frac{(k-1) \left( \frac{q}{f} \right)}{(k+1)} \right) \cdot \left( \frac{2q - \frac{c}{f}}{(p+0.1) (k+1)} \right) x 10^{-3}
\]

\[
= 9 \times 0.4 \left( \frac{36 + 1.4}{4} \right) \cdot \left( \frac{36 - 1.4}{0.15 \times 4} \right) x 10^{-3}
\]

\[
= 3 \text{ m}
\]

This is the order of the strata movement actually observed.

In weak strata such as that found at Abernant it will be
noted that the stress strength of \( \frac{q}{f} \) is not highly significant,
provided its value is small. The most significant parameter
is the triaxial stress factor \( k \).
(b) Drivage Stability at Dawdon

A 4 m diameter drivage was made at Dawdon Colliery in strong siltstone at a depth of 480 m. The measured laboratory strength of the siltstone was 70 MPa, the triaxial stress factor, 3.5. Visual inspection showed the siltstone to be almost completely free of joints or bedding. An in-situ factor $f = 3$ is therefore appropriate. The drivage was supported by complete steel rings with an estimated resistance of 0.2 MPa.

Substitution in Equation 3.6.2 gives

$$c = 4 \times 0.06 \left( \frac{30 + 23.3}{4.5} \right) \left( \frac{24 - 23.3}{0.3 \times 4.5} \right)^{\frac{44}{25}} \times 10^{-3}$$

$= 2 \text{ mm}$

In the event, no strata movement was discernible, the roadway remaining in perfect condition throughout the siltstone.

In this case the strata strength term $\frac{c}{f}$ does influence the outcome, but, the movements in such instances being small, the value of the term is not important. A slight increase in $\frac{c}{f}$ gives a negative value to the second bracketed item, implying that strata failure would not then occur, and no yield zone would develop.

(c) General Application to Weak Strata Conditions

In coal mining practice it is frequently necessary to drive access roadways to follow an inclined seam. The strata around the roadways will not vary appreciably, but the depth of cover will increase progressively. The example given below shows how the equations developed can be used to predict the possible behaviour of the strata and the lining.
Consider a roadway of 6 m driven diameter, in weak Coal Measure strata. The following average properties have been assumed. They are based on the strata conditions which surround the Barnsley Seam in the new Selby Mine Project:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laboratory strength $\sigma$</td>
<td>40 MPa</td>
</tr>
<tr>
<td>In-situ strength factor $f$</td>
<td>5</td>
</tr>
<tr>
<td>Triaxial stress factor $k$</td>
<td>3</td>
</tr>
<tr>
<td>Density of coal measures $\gamma$</td>
<td>0.025 MN/m$^3$</td>
</tr>
</tbody>
</table>

A yield zone will not develop if $\bar{r} < r_o$ in Equation 3.3.9. If $\bar{r} = r_o$, then as $q = \gamma H'$, we have

$$2\gamma H' - \frac{\sigma}{f} + p' (k + 1) = (p + p') (k + 1)$$

and

$$p' = \frac{1}{2\gamma} \left( (p + 0.1) (k + 1) + \frac{\sigma}{f} \right)$$

assuming $p' (k+1)$ small and $p' = 0.1$ MPa

Substituting the requisite values gives

$H' = 176$ m with a lining strength $p = 0.1$ MPa

and $H' = 248$ m with a lining strength $p = 1.0$ MPa

From Equation 3.7.2, the minimum support $p_r$ to hold up the roof will be

$$p_r = d \frac{\gamma \sqrt{k}}{2 (k - 1)} = 0.1 \text{ MPa}$$

With these limitations in mind, a table making use of Equation 3.6.2 can be drawn up. This gives the closure to be expected at a given depth with a given lining resistance.
Table 1. Estimated Drivage Closure in Soft Strata (mm)

<table>
<thead>
<tr>
<th>Depth of cover (m)</th>
<th>Lining resistance (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>200</td>
<td>7</td>
</tr>
<tr>
<td>300</td>
<td>40</td>
</tr>
<tr>
<td>400</td>
<td>80</td>
</tr>
<tr>
<td>500</td>
<td>140</td>
</tr>
<tr>
<td>600</td>
<td>220</td>
</tr>
<tr>
<td>700</td>
<td>310</td>
</tr>
<tr>
<td>800</td>
<td>420</td>
</tr>
<tr>
<td>900</td>
<td>540</td>
</tr>
<tr>
<td>1000</td>
<td>680</td>
</tr>
<tr>
<td>1100</td>
<td>830</td>
</tr>
<tr>
<td>1200</td>
<td>1000</td>
</tr>
</tbody>
</table>

Reinforced concrete will only take a compressive strain of 1000 to 2000 microstrain before failure, the exact point at which failure occurs depending on the quality of the concrete. This gives an average figure of about 1.5 mm/m, or 9 mm for a roadway diameter of 6 m. Hence, if the concrete is to be poured directly against the strata at the end of the drivage, it may only be used in Section A of the Table. Used otherwise damage will occur when the roadway is driven further ahead. This type of failure was seen in the insets of Abernant Colliery [6] and of Wolstanton Colliery [32].
Concrete panel linings can be designed to give strengths of up to 1.0 MPa, with overall closures of 200 mm, by inserting compression pieces between the segments. Such linings would be acceptable in Section B of the Table. Concrete panel linings have proved very successful in the deep, but weak, Coal Measures of the Campine Coalfield [33].

Installation of immediate, permanent supports will be practically impossible in strata of the depth and lining resistance categorised in Section C of the Table. Under these conditions the recommended procedure is to install temporary supports, to wait until the movement has settled, and then to trim the roadway and install the permanent lining. Alternatively, in the case of coal mining, a development face can be taken and the roadway driven in the de-stressed area of the waste.

3.10 Conclusions

A formula has been developed which gives values in general agreement with the known phenomena associated with roadway closure in soft rocks at depth. Further investigation is required on some of the parameters to be inserted, such as in-situ strength, the expansion factor in the yield zone and the cohesive effect of broken material under pressure. Even with these parameters approximated, the formula gives more reliable information than the previously-adopted formula used out of context.