CHAPTER FOUR

DISTRIBUTION OF STRESS AND STABILITY OF PILLARS
DISTRIBUTION OF STRESS AND STABILITY OF PILLARS

4.1 Introduction

No satisfactory means of analysing the distribution of stress in a coal ribside adjacent to an excavation has been developed. Nevertheless, some estimate is required if help is to be given in the siting of roadways and in the determination of adequate pillar sizes. This is of particular importance in the planning of new mines, especially if retreat faces are involved, as extensive roadway drivage must be planned and executed before experience of the effects of face extraction is gained.

In order to obtain an approximation, one method of approach is to use the so-called 'stress balance'. As the total aggregate downward force remains that of the cover load, any stress rise over the ribside must be compensated for by an equivalent stress reduction over the waste, and vice versa. Knowledge of one can lead to an estimate of the other, provided the general shape of the stress distribution is known.

As a starting point, it is convenient to examine the probable stress deficiency in the waste, and, by calculating the area of the stress-distance diagram, to postulate the size and extent of the stress rise over the ribside. Application to a number of cases where the behaviour of the strata is known will show if the estimates are borne out in fact.

4.2 Stress Rise in the Waste Area

Actual measurements of the stress rise in the waste area have not been possible, mainly because of the difficulty of maintaining leads to the stress-meters buried in the waste.
Stress measurements in the pack will not suffice, since the filling of the void by a concentration of material causes a local, rapid stress rise as the roof and floor come together. The ultimate stress value reached in the pack depends on the strength of the floor; because the floor stratum is free to flow into the open roadway, this ultimate stress may be substantially less than cover load. Deductions as to the manner of stress rise in the waste can, however, be made from studying the roadway behaviour.

Numerous studies of convergence in a variety of roadways have been made and the results published. All exhibited a rapid rate of convergence near the face line, a rate which diminishes the further the roadway is from the face, until the final stabilised roadway height is reached. It is usually possible to fit a logarithmic or exponential type of equation to the closure curve.

Figure 16a shows a typical curve obtained from roof measurements at Babbington Colliery [34]. It is particularly relevant, as it was obtained in a centre gate road between two areas of extraction. A curve of the form

\[ w = K_1 \ln \frac{B}{B-C} \]

where \( w \) = distance from the face line
\( c \) = amount of roof lowering
\( K_1 \) and \( B \) are constants
\( \ln \) is the Napierian logarithm

gives a close approximation to the measured values. Although it was never actually measured, it is probable that closure in the waste remote from the roadway is also of this form. In this
a. Lowering of centre gate roof at Babbington Colliery

\[
v = 47.9 \ln \frac{0.64}{22.4} - e
\]

b. Compaction of broken shale from Hucknall Colliery

\[
\sigma = 431 \ln \frac{22.4}{22.4} - e
\]

**FIGURE 16. FORM OF CURVES FOR LOWERING OF ROOF AND COMPACTION OF BROKEN ROCK**
latter instance, however, the ultimate closure $W$ will be approximately equal to the height of coal extracted $M$. Hence in the waste we have

$$w = K_1 \ln \frac{M}{M-c} \quad \ldots (4.2.1)$$

In a caved waste, the roof lowers onto broken material, which gradually compacts. The compaction of broken material has been studied by a number of observers, and a curve of the form

$$\sigma = Ke^{(-b/c)}$$

is usually adopted [36] [36]. This gives a good fit, except at low values of applied stress. If broken material is considered to be capable of compacting back to its unbroken volume at very high stress, and if the rate of compaction is assumed to be proportional to the degree of compaction remaining, then an alternative form of the curve is

$$\frac{dc}{dc} = \frac{1}{K_2} (B-c)$$

i.e. $$\frac{d\sigma}{dc} = \frac{K_2}{B-c}$$

and, by integration,

$$\sigma = K_1 \ln \frac{B}{B-c}$$

Figure 10b shows this form of curve applied to results obtained by Price and May [36] for a granulated shale from Hucknall Colliery. The fit is good, and has the advantage of being correct even at low values of stress. If a horizon is considered in the upper roof above the caving height, and if the caved material is assumed to compact back to its original
unbroken volume at very high stress, then the ultimate closure B
will again be equal to the height of coal extracted M, and
\[ \sigma = K \ln \frac{M}{2} \frac{M-c}{M-c} \quad \text{... (4.2.2)} \]

The probable form of the stress rise in the waste plotted
against distance back from the face can now be found by dividing
Equation 4.2.2 by Equation 4.2.1.

\[ \frac{\sigma}{w} = \frac{K}{K_1} = \text{a third constant} \]

The two logarithmic curves thus combine to give a stress rise
with distance of linear form. It is not unreasonable to assume
that the stress rise back from a ribside will have a similar
linear form.

The probable form of stress rise in the waste having been
assessed, it is now necessary to determine the value to which
it will rise. Although the existence of rear stress abutments
has often been postulated [37] [38] [39], no definite evidence
as to their existence has been found. Nor are rear stress
abutments necessary to satisfy the equilibrium conditions. In
the elastic condition, a face can be considered as an elongated
ellipse. For an opening circular in shape, the tangential stress
will be equal all around. As the opening elongates into an
ellipse, the tangential stress will rise at the sides, but will
decrease at the top and bottom. If the ellipse elongates
sufficiently, the top and bottom will begin to touch, and
a vertical stress between them will be gradually generated,
this stress eventually reaching but not exceeding cover load.

Alternatively, the ground above the waste can be considered
as a thick but pliable beam cantilevering out from the ribside.
A short beam will bend, but will not 'bottom' on the floor. If the beam is long enough, the end will just touch the floor, but no force will be generated. As the beam is extended it will gradually sit on the floor until, at a point sufficiently far from the ribside, the loading on the floor will equal the distributed dead weight of the beam. In static equilibrium terms, although the total force up must continue to equal the total force down (ie the previously-mentioned 'stress balance' must be maintained), the turning moment produced by the stress increase over the ribside and the stress decrease in the waste will be balanced by internal forces generated in the beam (the bending moment). The existence of a rear abutment is not required for equilibrium.

Because of the difficulties of stress measurement in the waste, the distance back of the return to cover load has not actually been measured. However, in their research as to the width of the so-called 'pressure arch', a team of investigators in the north east of England [40] made detailed observations of a large number of roadways in their efforts to determine the extent of the reduction of stress below cover load. The findings of this team are shown in Figure 17, a look at which will show that, in large part, the plotted values fall in a band between 0.2 H and 0.3 H, where H is the depth below the surface. As the ultimate purpose of this section is to deduce the stress in the ribside as an aid to the siting of roadways, the higher value of 0.3 H will be adopted, since this will give the greater stress and hence will err on the side of safety.
FIGURE 17. OBSERVED WIDTHS OF MAXIMUM PRESSURE ARCH (BASED ON VALUES FROM REF. 40)
Using a distance of 0.3 \( H \) for the return to cover load distance implies a critical width of extraction at seam level of 0.6 \( H \). This is in contrast to the 1.4 \( H \) critical width found to produce maximum subsidence at the surface [41]. The difference requires some comment.

It has been claimed that subsidence at intervening horizons below the surface can be deduced by assuming a 'free surface' to exist at the horizon in question [42]. This concept has been used, in conjunction with the Subsidence Engineers' Handbook [41], as the means of plotting Figure 18, which shows intermediate subsidences for a face 200 m long at a depth of 500 m. Evidence by which the claim can precisely be justified is not strong, but it does imply that a major vertical extension of the strata occurs as the face horizon is approached in the neighbourhood of the ribside. Such major extension has been observed in practice [43][44], and is undoubtedly present. This extension will result in a greater closure rate at seam level than is implied by surface subsidence. Hence acceptance of a critical width of 0.6 \( H \) at seam level is not incompatible with the 1.4 \( H \) critical width of surface.

As is evident from the foregoing arguments, it is probable that the reduction of stress below cover load in an extensive waste greater than 0.6 \( H \) across can be approximately represented by a triangular stress distribution, beginning from zero at the ribside and rising to cover load \( H \) in a distance of about 0.3 \( H \), as is shown in Figure 19a. If the waste is less than 0.6 \( H \) across, the stress in the centre will not reach cover load, and the stress reduction associated with each ribside will be trapezoidal in shape, as is shown in Figure 18b.
FIGURE 18. SUBSIDENCE LINES FOR 200 m FACE AT DEPTH OF 500 m
a. Face Length > 0.6 H

b. Face Length < 0.6 H

\[ \frac{3}{4} = \frac{\text{Cover Load}}{yH} \]

FIGURE 19. PROBABLE STRESS REDUCTION BELOW COVER LOAD IN THE WASTE
Reference Figure 19a, the area of the stress-distance deficiency will be

\[ \frac{1}{2} \cdot (0.3 \, H) \cdot (\gamma H) \]

ie, for \( W > 0.6 \, H \), \( A = 0.15 \, \gamma H^2 \) \quad \ldots(4.2.3)\\

Reference Figure 19b, by taking equivalent triangles, the stress reduction \( d \) at the centre of the face will be

\[ d = \frac{\gamma H}{0.3 \, H} \left(0.3 \, H - \frac{W}{2}\right) \]

\[ = \gamma \left(H - \frac{W}{0.6}\right) \]

and the area of the stress-distance diagram will be

\[ 0.15 \, \gamma H^2 - \frac{1}{2} \left(0.3 \, H - \frac{W}{2}\right) \gamma \left(H - \frac{W}{0.6}\right) \]

ie, for \( W < 0.6 \, H \), \( A = \frac{1}{2} \, \gamma W (H - \frac{W}{1.2}) \) \quad \ldots(4.2.4)\\

where \( W = \) length of face

\( H = \) depth of cover

\( \gamma = \) average density of the strata
4.3 Stress Rise in the Yield Zone

If a longwall extraction is likened to an extended crack, theoretically there would be infinite stress in the abutment. Soft rock will not take a high stress near a boundary, and a yield zone will develop (see Section 3.2). The failure criterion will again be \( \sigma_x = k_0 \sigma_y \), with \( k \) the triaxial stress factor and \( \sigma_y \) and \( \sigma_x \) the principal stresses.

Where the seam in question can be considered as a relatively weak stratum sandwiched between a strong roof and floor, with the yield taking place preferentially in the weak stratum, the increase of pressure into the yield zone can be shown to be exponential in form. An approximate solution was published by the author in 1972 [5], but Airey [45] has since been able to produce a more rigorous solution. If \( p' \) is considered small compared to \( \sigma_y \), the Airey solution may be written as

\[
\sigma = \frac{p}{\lambda} + \frac{p'}{\lambda} \quad ; \quad \sigma_x = \frac{1}{k} \sigma_y
\]

where \( \sigma_y \) and \( \sigma_x \) are the principal stresses at the centre line of the weak stratum,

\( p \) = horizontal pressure supplied by the ribside supports,
\( p' \) = the unidirectional compressive strength of the failed rock (see Section 3.5),
\( k \) = triaxial stress factor,
\( x \) = distance from ribside,
\( M \) = height of the seam extracted,
\( F \) = a function of \( k \),

\[
F = \frac{k-1}{\sqrt{k}} + \left( \frac{k-1}{k} \right)^2 \tan^{-1} \sqrt{\frac{k}{k}} \quad \text{with} \quad \tan^{-1} \sqrt{\frac{k}{k}} \text{ in radians.}
\]
When the roof, seam and floor are all soft, yield will also occur in the roof and floor, and the rate of stress rise will hence be lower. At the most it cannot exceed that deduced previously (see Section 3.3) for a circular roadway in homogeneous rock, as one side of the circle will now be open and the rock here will not be contracting on itself. In the absence of a more accurate solution, Equations 3.3.1 and 3.3.2 will be accepted as applying, the radius \( r \) of the roadway being replaced by the half-height of extraction, \( \frac{M}{2} \). If \( p' \) is considered small compared to \( \sigma \), then

\[
\sigma = k(p + p') \left( \frac{x + \frac{M}{2}}{\frac{M}{2}} \right) ; \quad \cdots (4.3.2)
\]

\[
\sigma = \frac{1}{k} \sigma_r
\]

where \( x \) is again the distance from the ribside.

The stress in the yield zone will not rise indefinitely. Measurements made in the past [5] suggest that the abutment peak reached will be of the order of three to five times cover load. There is a possible explanation of this. As the lateral constraint increases in the yield zone, a value will eventually be reached which equals the original virgin horizontal stress \( q \) (taken as equal to the original virgin vertical stress = \( \gamma H \)). It need not rise further to establish equilibrium with the elastic zone beyond. At this value of retaining pressure, the failure criteria will be

\[
\sigma = kq + \sigma_0 \text{ in the elastic zone } \cdots (4.3.3)
\]

\[
\sigma_r = kq \text{ in the yield zone } \cdots (4.3.4)
\]
which probably represents the conditions at the yield/elastic boundary. The step in value occurs because the rock suddenly loses cohesion on failure. For representative values for Coal Measured strata, Equation 4.3.3 gives values of between three and five times cover load, and both Equations 4.3.3 and 4.3.4 will be accepted as hypothetically correct in the deductions which follow.

The width $x_b$ of the yield zone and the area $A_b$ below the corresponding stress curve can now be calculated for the two cases mentioned above.

(a) **Yield in seam only**

From Equations 4.3.1 and 4.3.4 we have

$$
\sigma_y = kq = (p + p') \frac{k}{b_m} 
$$

$$
\therefore x_b = \frac{M}{F} \ln \left( \frac{q}{p + p'} \right) 
$$

$$
A_b = \int_{0}^{x_b} \sigma_y \, dx = \frac{M}{k} \left( q - (p + p') \right) 
$$

(b) **Yield in roof, seam and floor**

From Equations 4.3.2 and 4.3.4 we have

$$
\sigma_y = kq = k \left( p + p' \right) \left\{ \frac{x}{2} \frac{k}{M} \right\} 
$$

$$
\therefore \quad x_b = \frac{M}{2} \left\{ \left( \frac{q}{p + p'} \right)^{\frac{1}{k-1}} - 1 \right\} 
$$

$$
A_b = \int_{0}^{x_b} \sigma_y \, dx = \frac{M}{2} \left( p + p' \right) \left\{ \left( \frac{q}{p + p'} \right)^{\frac{1}{k-1}} - 1 \right\} 
$$
4.4 Stress Decay in the Elastic Zone

Beyond the yield/elastic boundary, the stress concentration will decay, until it eventually reaches the cover load. Whether this decay will follow a power law, exponential law or some other law is not known, but an exponential law has been assumed in order to facilitate the calculation. This implies that, at any point on the curve, the rate of stress decrease is proportional to the amount of stress outstanding above the cover load.

The curve will commence at $\hat{\sigma}$ and be asymptotic to the cover load $q$. This requires an equation of the form

$$(\sigma - q) = (\hat{\sigma} - q) \exp\left(\frac{x - \chi_b}{C}\right) \quad \ldots (4.4.1)$$

where $x$ is the distance from the ribside, $C$ is a constant having the units of distance and $\exp(F) = e^F$. The area $A_s$ above the cover load line, as shown in Figure 20, will then be

$$A_s = \int_{x_b}^{\infty} (\sigma - q) \, dx$$

$$= C (\hat{\sigma} - q) \quad \ldots (4.4.2)$$

The value of the constant $C$ can be determined by equating the total area deficiency below the cover load line to the total area augmentation above it.

Referring to Figure 20,

$$A_v + A_2 = A_3 + A_s$$

By adding $A_2$ to both sides of the equation, we obtain
\[ \dot{\epsilon} = kq + \sigma_o \]

\[ \sigma_y = kq \]

\[ \sigma = (\sigma - q) e^{-\frac{x_b}{\sigma}} + q \]

\[ A_2 + A_3 = A_B \]

\[ A_B \approx \gamma H \]

\[ q = \gamma H \]

FIGURE 20. IDEALISED STRESS AREA BALANCE ACROSS RIBSIDE
\[ A_y + (A_1 + A_2) = (A_2 + A_3) + A_z \]

\[ \therefore A_y + q \cdot x_b = A_b + A_z \]

From Equation 4.4.2, \( A_z = C (\sigma - q) \)

\[ \therefore C = \frac{A_y + q \cdot x_b - A_b}{\sigma - q} \quad \text{...(4.4.3)} \]

Formulae for \( A_y, x_b, A_b \) and \( C \) for the various conditions have already been established in Equations 4.2.3 or 4.2.4, 4.3.5 or 4.3.7, 4.3.6 or 4.3.8 and 4.3.3. Substitution of these values as well as \( q = \gamma H = \frac{H}{40} \), also assuming \((p + p')\) to be small compared to \( q \), gives:

(a) For yield in seam only

\[
W > 0.6 \, H, \quad C = \frac{0.15H + x_b - \frac{M}{F} \frac{k}{H}}{(k-1) + 40 \frac{\sigma}{H}} \quad \text{...(4.4.4)}
\]

\[
W < 0.6 \, H, \quad C = \frac{W}{2} \left(1 - \frac{W}{1.2H}\right) + x_b - \frac{M}{F} \frac{k}{H}
\]

\[ (k-1) + 40 \frac{\sigma}{H} \quad \text{...(4.4.5)} \]

(b) For yield in roof, seam and floor

\[
W > 0.6 \, H, \quad C = \frac{0.15H - \frac{M}{2}}{(k-1) + 40 \frac{\sigma}{H}} \quad \text{...(4.4.6)}
\]

\[
W < 0.6 \, H, \quad C = \frac{W}{2} \left(1 - \frac{W}{1.2H}\right) - \frac{M}{2}
\]

\[ (k-1) + 40 \frac{\sigma}{H} \quad \text{...(4.4.7)} \]

where \( H = \text{depth of cover (m)} \)

\( W = \text{width of extraction (m)} \)
\( k = \text{triaxial stress factor} \)
\( \sigma_0 = \text{unidirectional compressive strength in situ (MPa)} \)
\( x_b = \text{width of yield zone (m)} \)
\( M = \text{height of extraction (m)} \)

Hence the complete stress diagram can be approximated.

The hypothetical stress balance for a ribside adjacent to a wide excavation, assuming typical values for \( H, k, M, \sigma_0 \) and \( p \), is shown in Figure 21.

4.5 Required Widths of Long Protection Pillars

A roadway placed in the highly-stressed area of the ribside will suffer damage. As the assumed stress decay curve is asymptotic to the cover load, theoretically it is not possible to go beyond the area of increased stress. Most roadways will tolerate a small increase in stress, however, without suffering undue damage, and it is now a matter of deducing a logical position for the roadway in terms of the stress-distance diagram.

If the exponential decay curve is replaced by a triangular stress distribution of equal area in terms of the stress-distance diagram, then the 'stress balance' will be maintained in the presentation. If the roadway is then placed a further distance \( x_b \) away, where \( x_b \) is the width of the yield zone which would develop if an excavation were carried out beyond the roadway, then the roadway would lie in an area where the stress was not much above cover load. The advantage of introducing the extra distance \( x_b \) will become apparent later.
If, as is shown in Figure 21, b is the base of the
equivalent triangle, then

\[ \frac{1}{2} b (\delta - q) = C (\delta - q) \]

\[ \therefore \quad b = 2C \]

Hence a roadway should not be placed nearer to a ribside than
2 \( (C + x_b) \), which fixes the size of the barrier pillar required
in order to give protection to a roadway from an excavation
beyond. The position of the roadway is also shown in Figure 21.

In retreat mining, finger pillars are frequently left between
the faces to protect the roadways. The roadways are pre-driven,
and the widths of the intervening pillars must be planned in
such a way that, when one face is taken, the roadway of the
next face is not seriously affected. Hence, \( 2 (C + x_b) \) fixes
the required size of pillars in finger extraction. If the
equivalent triangular stress distribution is assumed, then,
following the extraction of both flanking faces, the stress
triangles will add together and the pillar of minimum required
width will have an elastic centre core with an even stress
distribution equal to \( \delta \) right across. Flanking the core on
both sides will be yield zones, which retain the centre core
in the elastic state.

If the average stress in the core were to rise above \( \delta \),
the pillar would not fail, but the widths of the yield zones
would have to increase beyond that originally established when
the adjacent faces were won. In consequence, the roadways will
suffer damage because of the additional strata expansion.
This phenomenon is sometimes referred to as instability in the pillar; therefore

\[ Q = 2 \left( C + x_b \right) \]  \hspace{1cm} \ldots(4.5.1)

also fixes the minimum width of a 'stable' pillar.

In practice the centre core may not have an even stress right across. Measurements made by stresometers have indicated stress peaks within the elastic zone [46] [47]. These peaks may be related to the so-called 'Weber waves' [37]. The average stress across the pillar, however, has still been found to be that deduced by the 'energy balance'.

4.6 Required Size of Rectangular Pillars

Variations of the above method of assessing pillar size may also be applied to rectangular pillars. Lines of such pillars are frequently formed in the USA, where there is the legal requirement that development roadways be driven as interconnected multi-entry systems. A typical example of this system is shown in Figure 22.

In the long continuous pillar, the stress-distance diagram can be treated two-dimensionally and the areas above and below the cover load line brought into balance. With a rectangular pillar a three-dimensional concept must be considered, involving volumes below the stress envelope. In this case it is more convenient to take the total volume above the zero stress plane, which represents the total load on the pillar, and balance the total load which the pillar will take to the total load imposed upon it.
FIGURE 22. TYPICAL MULTI-ENTRY SYSTEM (USA)

Area of cover resting on each pillar
Figure 23 shows the hypothetical stress envelope above a rectangular pillar of length P and width Q at the probable limit of stability. The precise volume of the stress-distance diagram will be

\[ V = \int_0^r (P - 2x) (Q - 2x) \, dx + c (P - 2x_b) (Q - 2x_b) \]

where x is a function of c obtainable, depending on the circumstances, either from Equation 4.3.1 or from Equation 4.3.2. The integral can be solved, but this leads to expressions of such complexity that their use is impracticable, particularly in view of the other approximations already made.

In the course of the analysis, the values of x_b and A_b as per Equations 4.3.5 to 4.3.8 will probably already have been obtained. Use can be made of these, and a close approximation to the volume required can be obtained by adding the volume of the 'core' to the cross-sectional area A_b of the yield zone multiplied by its mean perimeter. If this volume (or load) is represented by \( L_R \), then

\[ L = \frac{c}{R} (P - 2x_b) (Q - 2x_b) + 2 A_b (P + Q - 2x_b) \quad (4.6.1) \]

The total load imposed on the pillar will be the dead weight of the ground above, over the area indicated in Figure 22. On each pillar this will be

\[ L = \gamma H (P + w) (Q + w) \quad (4.6.2) \]

In a given instance the length P will usually be fixed, and the required width Q, which defines the distance between the roadways, can be found by equating Equations 4.6.1 and 4.6.2.
FIGURE 23. HYPOTHETICAL STRESS ENVELOPE ABOVE A RECTANGULAR PILLAR AT THE LIMIT OF STABILITY
Frequently, roadways are of much greater width than height. Such being the case, it is probable that the stress abutment on each side will have developed to the full $\hat{\sigma} = kq + \sigma_o$ (Equation 4.3.3). Occasionally, however, because of ventilation or transport requirements, this is not the case, and the roadway will be somewhere between the equal width-height condition of Equation 3.3.5 and the wide extraction condition of Equation 4.3.3. Some means of interpolating between these two limits is required.

If an elliptical opening in an elastic medium in a uniform hydrostatic stress field $q$ is considered, then it can be shown [48] that the tangential stress at the ends of the major axis is $2Rq$ and at the ends of the minor axis $\frac{2}{R}q$, where $R$ is the ratio of the major to the minor axis. This is illustrated in Figure 24a. If yield occurs, the stress concentration (i.e. the abutment peak) will move away from the boundary of the opening and reduce to

$$\hat{\sigma}_0 = \frac{k (2q - \sigma_o)}{k + 1} + \sigma_o \quad \text{(see Equation 3.3.5)}$$

$$= 2 \frac{k}{k + 1} q + \frac{\sigma_o}{k + 1}$$

If this can be considered to be the 'norm' for an opening the width of which is equal to its height, then the stress peak $\hat{\sigma}_m$ adjacent to a rectangular opening can be approximated by

$$\hat{\sigma}_m = \left\{ \frac{k (2q - \sigma_o)}{k + 1} + \sigma_o \right\} R \quad \ldots \text{(4.6.3)}$$

where $R = \text{ratio of width to height}$

$q = \gamma H$
FIGURE 24. TANGENTIAL STRESS PEAK ALONGSIDE OPENINGS OF RESTRICTED WIDTH
up to the limit when \( \hat{C} = kq + \sigma_c \). This is illustrated in Figure 24b, for typical values of \( H \) and \( \sigma_c \).

The corresponding values of width of yield zone \( x_0 \) and stress-distance area \( A_0 \) can be found by substituting from Equations 4.3.5 to 4.3.8 a modified value \( q_m \) for the hydrostatic stress field \( q \). In Equations 4.3.5 and 4.3.7, \( \beta \) will now equal \( \frac{\hat{C} - \sigma_c}{k} \).

\[ q_m = \frac{\beta - \sigma}{k} \quad \ldots \quad (4.6.4) \]

This method will provide a reasonable approximation if the roof, seam and floor are comparable, but only a rough approximation when there is a weak stratum between a strong roof and floor.
4.7 Application of Formulae to Specific Cases

How to use the formulae developed for the distribution of stress and for stability of pillars is best seen by applying them to specific cases. Where possible, situations have been chosen which give some indication of the success or otherwise of the predictions made. For easy reference, a summary of the more important formulae developed and the nomenclature used is given in Appendix 4.

(a) Stress Prediction ahead of a Longwall Face

In the course of development of the MRE Stressmeter [1], the change of stress ahead of a longwall retreatng face was measured at Bates Colliery in Northumberland [49]. The stressmeters were placed at the ends of long boreholes drilled parallel to and well in advance of the face, and both the stress change and the convergence of the roadway ahead of the face were measured as the face approached. These results can be compared to the hypothetically predicted stress change.

**Known information:**

- height of face extraction, \( h = 1.32 \text{ m} \)
- depth of cover, \( H = 229 \text{ m} \)
- support resistance against face, \( p = 0 \)

**Typical information assumed:**

- allowance for cohesion of failed coal, \( p' = 0.1 \text{ MPa} \)
- triaxial stress factor for coal, \( k = 4 \)
- laboratory strength of coal, \( \sigma = 25 \text{ MPa} \)
- reduction factor for in situ strength, \( f = 5 \)
The roof and floor were mudstone, therefore yield probably occurred in roof, seam and floor.

Hence:

strength of strata in situ, \( c_o = \sigma f = 5 \) MPa

cover load, \( q = \gamma H = 5.7 \) MPa

abutment peak, \( \hat{c} = kq + c_o = 27.9 \) MPa

\[ (Equation\ 4.3.3) \]

width of yield zone,

\[ x_b = \frac{M}{2} \left( \frac{q}{p + p'} \right)^{-1} \]

\[ = 1.88 \text{ m} \] (Equation 4.3.7)

exponential decay factor, \( C = \frac{0.15H - \frac{3M}{k-1}}{40 \frac{c_o}{H} + 1} \)

\[ = 8.7 \text{ m} \] (Equation 4.4.6)

stress above cover load in yield zone,

\[ (c - q) = k(p + p') \left\{ \frac{x + \frac{3M}{k-1}}{x} \right\}^{k-1} - q \]

\[ = 0.4 \left\{ \frac{x + 0.66}{0.66} \right\}^3 - 8.7 \] (Equ 4.3.2)

stress above cover load in elastic zone,

\[ (c - q) = (\hat{c} - q) \exp \left( \frac{x - x_b}{c} \right) \]

\[ = 22.2 \exp \left( \frac{1.88 - x}{8.7} \right) \] (Equ 4.4.1)

where \( x \) is the distance from the face line and \( \exp (F) = e^F \).

The above information allows the hypothetical stress above cover load to be plotted—see Figure 25, in which it is compared
FIGURE 25. MEASUREMENTS AHEAD OF 10 WEST FACE AT BATES COLLIERY
to the measured values from the two stressmeters and to the roadway convergence. The stressmeters probably give good qualitative information, but quantitively they are not too reliable; hence the close agreement with the readings from stressmeter Plug 1 is probably no more than coincidental. However, both these stressmeters, and others installed ahead of a neighbouring retreat face, all began to show a stress rise about 30 m ahead of the face. This was confirmed by the onset of roadway convergence. Hence the agreement with the hypothetical prediction is good.

(b) Application to Roadway Convergence Results

Whittaker and Singh have published measurements of closure in roadways protected by pillars of various widths [50] and have drawn a statistically-derived curve through their results. This is shown in Figure 26. Since the amount of roadway closure will be associated with the degree of stress produced by the adjacent face, their curve should be similar in shape to the hypothetically- deduced stress curve.

Following discussions with Whittaker and Singh, typical average values for the conditions they investigated were chosen. These were:

- depth of cover, \( H = 500 \text{ m} \)
- length of face, \( W = 230 \text{ m} \)
- roadway height, \( M = 3 \text{ m} \)
- support, \( p + p' = 0.1 \text{ MPa} \)

Many of the measurements were associated with soft roofs and floors similar to those encountered in the Blackshale Seam.
FIGURE 26. STATISTICAL ROADWAY CLOSURE CURVE OF WHITTAKER AND SINGH WITH HYPOTHETICAL STRESS CURVE SUPERIMPOSED
at Hucknall Colliery. Measurements made at MRDE gave the average triaxial stress factor of this seam as \( k = 3.3 \); the in situ strength of the strata is small compared to the strata stresses at this depth.

From this data the following values were calculated:

- cover load, \( q = \gamma H \) = 12.5 MPa
- abutment peak, \( \sigma = kq + \sigma_0 = 41.25 \) MPa (Equ 4.3.3)
- width of yield zone, \( x = \frac{M}{2} \left( \frac{q}{p + q} \right)^{\frac{1}{2}} \) \( = 10.7 \) m (Equation 4.3.7)
- length of face/depth \( \frac{W}{H} = 0.46 \), i.e. \( W < 0.6 \) H

\[ \text{exponential decay factor } C = \frac{W}{2} \left( 1 - \frac{W}{1.2H} \right) - \frac{M}{2} \]

\[ = 30.2 \text{ m (Equation 4.4.7)} \]

- and stress \( \sigma = (\sigma - q) e + q \) (Equ 4.4.4)

\[ \frac{12.7 - x}{30.2} \]

\[ = 28.75 e^{\frac{12.7 - x}{30.2}} + 12.5 \text{ MPa} \]

where \( x \) is the width of the pillar (m).

These values allow the hypothetical stress curve to be drawn, and it is shown superimposed on the diagram of Whittaker and Singh (Figure 26). The agreement is close. The hypothetical minimum pillar width for stability will be \( Q = 2 (C + x_p) = 82 \) m, which is also shown.
The data presented by Whittaker and Singh involved various depths of cover and various lengths of face. Corrections can be made to accommodate these variations to a standard depth and a standard length of face. Take as an example the sites in the Blackshale Seam, details of which were tabulated in the publication in question [50] and are reproduced in the first five columns of Table 2. The above method was used to calculate the hypothetical stress for the various values of $H$, $W$ and $x$ given, and are shown in the sixth column. Similar values for $H = 500$ m and $W = 230$ m, but using the relevant value of $x$, were calculated and are shown in the seventh column. These were used to modify the measured values of $\frac{C}{h}$ as indicated in the last column of Table 2.

The modified roadway closures have been plotted against pillar width in Figure 27. The head of the arrow indicates the modified value, the tail of the arrow the original measured value. The fit of the points to the hypothetical stress curve is improved. In the publication, site 4 was reported as having a 'successful inner ring'; at site 13 the face-side pack had been replaced by chocks. There were no comments about the other sites.

Figure 27 implies that a 'norm' for the roadway closure in the Blackshale Seam in Nottinghamshire can be obtained by multiplying the hypothetical stress by a constant,

\[ \text{i.e., } \frac{C}{h} \text{ (MPa)} = c \times 4.6 \]
### TABLE 2

Corrections to Measured Roadway Closure to Convert to a Standard Depth $H = 500$ m and Face Length $W = 230$ m

**Blackshale Seam, Nottinghamshire**

<table>
<thead>
<tr>
<th>Reference No. in Publication</th>
<th>Depth of Cover $H$(m)</th>
<th>Face Length $W$(m)</th>
<th>Pillar Width $x$(m)</th>
<th>Closure in Terms of seam Ext. of Height % $c$</th>
<th>Actual Hypothetical Stress $\sigma$(MPa)</th>
<th>Equivalent at Stress at $H = 500$ m $\sigma_s$(MPa)</th>
<th>Modified Roadway Closure $\sigma_c$</th>
<th>$%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>505</td>
<td>203</td>
<td>18</td>
<td>65</td>
<td>35.1</td>
<td>35.1</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>470</td>
<td>218</td>
<td>220</td>
<td>47</td>
<td>11.8</td>
<td>12.5</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>540</td>
<td>219</td>
<td>80</td>
<td>70</td>
<td>16.8</td>
<td>15.4</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>540</td>
<td>219</td>
<td>180</td>
<td>132</td>
<td>13.6</td>
<td>12.6</td>
<td>122</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>416</td>
<td>230</td>
<td>197</td>
<td>67</td>
<td>10.4</td>
<td>12.6</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>416</td>
<td>230</td>
<td>222</td>
<td>53</td>
<td>10.4</td>
<td>12.5</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>395</td>
<td>174</td>
<td>37</td>
<td>83</td>
<td>17.2</td>
<td>24.5</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>395</td>
<td>174</td>
<td>11</td>
<td>140</td>
<td>32.3</td>
<td>41.0</td>
<td>178</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>459</td>
<td>200</td>
<td>197</td>
<td>40</td>
<td>11.5</td>
<td>12.6</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>693</td>
<td>206</td>
<td>200</td>
<td>100</td>
<td>17.5</td>
<td>12.6</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>533</td>
<td>190</td>
<td>30</td>
<td>136</td>
<td>28.9</td>
<td>27.7</td>
<td>130</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 27. GATERoad Closure IN BlackSlate Seam, NOttingHamshire
(Point of Arrow shows Modified Value for h = 500 m, W = 230 m)
The value of the constant for other seams will depend on the type of strata. The Blackshale Seam has an abnormally low value of triaxial stress factor, and roadways formed in this Seam will close more readily than most.

(c) Fixing of Pillar Sizes and Comparison to 'One Tenth Depth plus 15 yd' Rule

The average laboratory strength of coal in Britain is 20 MPa, giving an approximate in situ value of about 4 MPa. In general, although the sides of an opening are well shielded, there is little positive resistance to sideways closure, and a value of \( p + p' = 0.1 \) MPa can be assumed. The average height of a roadway is about 3 m.

Hence, for pillars with wide excavations on both sides (in excess of 0.6 H), in typical British conditions,

\[
\begin{align*}
\text{width of yield zone, } x_b &= \frac{N}{2} \left( \frac{\frac{q}{p + p'}}{k-1} \right)^{-1} \quad \text{(Equation 4.3.7)} \\
&= 1.5 \left( \frac{0.25 H}{k-1} \right)^{-1} \quad \text{m}
\end{align*}
\]

\[
\begin{align*}
\text{exponential decay factor, } C &= \frac{0.15 H - \frac{q}{2M}}{(k-1) + \frac{0.25}{H}} \quad \text{(Equation 4.4.6)} \\
&= \frac{0.15 H - 1.5}{(k-1) + \frac{160}{H}} \quad \text{m}
\end{align*}
\]

\[
\begin{align*}
\text{minimum pillar width, } Q &= 2 \left( C + x_b \right) \quad \text{(Equation 4.5.1)}
\end{align*}
\]

For the average coal, the triaxial stress factor \( k \) varies from 3.5 to 4.0. Values of \( x_b \), \( C \) and \( Q \) for both these values of \( k \) were calculated for various values of \( H \), and are shown
tabulated in Table 3 and graphed in Figure 28. Very occasionally
values of \( k = 3 \) are also found, notably in the Barnsley Seam in
Yorkshire, and this lower limit of \( k \) has been shown for
completeness.

The graph of \( Q \) against \( H \) comprises almost straight lines,
and can be approximated by

\[
\begin{align*}
\text{for } k = 4.0, \quad & Q = 0.108 H + 2 m \\
\text{for } k = 3.5, \quad & Q = 0.131 H + 4 m \\
\text{for } k = 3.0, \quad & Q = 0.176 H + 6 m
\end{align*}
\]

A rule of thumb frequently used for fixing pillar sizes is
to make the width equal to 'one tenth the depth plus 15 yd', ie
\( Q = 0.1 H + 14 m \). This is shown by the dotted line in Figure 28.
For the average coal, the correlation with the hypothetical
pillar size is good.

At the deeper horizons, the width of extraction \( W \) is usually
less than 0.6 \( H \). If 200 m long faces are considered, this limit
will be at depth \( H = 200 / 0.6 = 333 m \). Below this depth, the
full cover load will not be reached at the centre of the caved
waste, and a reduced pillar size will be required as a
consequence. Hence, for a 200 m face below \( H = 333 m \),

\[
C = \frac{W}{2} (1 - \frac{W}{1.2 H}) \cdot \frac{M}{2}.
\]

\[\text{(Equation 4.4.7)}\]

\[
= \frac{100}{(k-1)} \left(1 - 200 \frac{1.2}{H} \right) - 1.5
\]

\[\text{m}\]

- 88 -
### Table 3

**Width of Yield Zones and Required Pillar Sizes**

<table>
<thead>
<tr>
<th>Depth of Cover</th>
<th>Width of Yield Zone, ( x_b )</th>
<th>Width of Pillar, ( Q ) (Wide Extraction)</th>
<th>Width of Pillar, ( Q ) (200 m Faces)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H = 4.0 )</td>
<td>( k = 3.5 )</td>
<td>( k = 3.0 )</td>
</tr>
<tr>
<td>(m)</td>
<td>(m)</td>
<td>(m)</td>
<td>(m)</td>
</tr>
<tr>
<td>100</td>
<td>2.9</td>
<td>3.9</td>
<td>6.0</td>
</tr>
<tr>
<td>200</td>
<td>4.0</td>
<td>5.7</td>
<td>9.1</td>
</tr>
<tr>
<td>300</td>
<td>4.8</td>
<td>6.9</td>
<td>11.5</td>
</tr>
<tr>
<td>400</td>
<td>5.5</td>
<td>8.0</td>
<td>13.5</td>
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<tr>
<td>500</td>
<td>6.0</td>
<td>8.9</td>
<td>15.3</td>
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<td>600</td>
<td>6.5</td>
<td>9.6</td>
<td>16.9</td>
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<td>700</td>
<td>6.9</td>
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<td>800</td>
<td>7.3</td>
<td>11.0</td>
<td>19.7</td>
</tr>
<tr>
<td>900</td>
<td>7.6</td>
<td>11.6</td>
<td>21.0</td>
</tr>
<tr>
<td>1000</td>
<td>8.0</td>
<td>12.2</td>
<td>22.2</td>
</tr>
<tr>
<td>1100</td>
<td>8.3</td>
<td>12.7</td>
<td>23.4</td>
</tr>
<tr>
<td>1200</td>
<td>8.5</td>
<td>13.2</td>
<td>24.5</td>
</tr>
</tbody>
</table>
FIGURE 28. ESTIMATE OF PILLAR SIZES REQUIRED BETWEEN WIDE EXCAVATIONS (COMPAIRED TO "ONE TENTH DEPTH PLUS 15 YD" RULE)
The modified values of Q are included in Table 3 and have been plotted in Figure 29. They imply that, in finger extraction, pillar width in the average coal seam need not exceed 70 to 90 m. This does not include such seams as the Blackshale in Nottinghamshire \((k = 3.3)\) and the Barnsley Seam in Yorkshire \((k = 3.0)\), where substantially wider pillars will be required at depth.

(d) **Pillar Size in the Shafton Seam at Riddings Drift**

In the Shafton Seam at Riddings Drift it was normal to leave pillars 22.8 m wide, separating areas of extraction 63.8 m wide. Under these conditions the pillars remained stable and no deterioration occurred in the roadways on the opposite side of the pillar. The height of the roadways was 1.68 m and the depth of cover 200 m.

The layout of the district necessitated the leaving of one pillar 18.3 m wide. The pillar split vertically in several places 3 m to 5 m from the ribside, considerable spalling of the roadway sides took place, and roof to floor convergence occurred. Hence the minimum pillar size for stability lay between 18.3 m and 22.8 m. This can be compared with the calculated value using the formulae.

Samples of coal taken and tested in the laboratory gave an average triaxial stress factor of \(k = 3.7\) and a unidirectional compressive strength of 26.6 MPa. Assuming a strength reduction factor of 5, this is equivalent to 5.3 MPa in situ. Used in the roadways was a goal-post type of support system, which did not make contact with the sides. Hence, on this occasion, \(p + p''\) was taken as equal to 0.05 MPa.
For $Q = 4$ MPa

$H = 3$ m

$p + p'' = 0.1$ MPa

**Figure 29. Estimate of Pillar Sizes Required Between 200 m Wide Faces**
From the above data we have

\[ H = 200 \text{ m} \]
\[ W = 63.8 \text{ m} \]
\[ M = 1.68 \text{ m} \]
\[ k = 3.7 \]
\[ c_0 = 5.3 \text{ MPa} \]
\[ p + p^* = 0.05 \text{ MPa} \]

\[ \therefore x_b = \frac{M}{2} \left\{ \frac{\frac{q}{W + p^*}}{\frac{1}{x_b + 1}} \right\}^{-1} = 3.8 \text{ m} \quad (\text{Equation 4.3.7}) \]

\[ \frac{W}{H} = \frac{63.8}{200} = 0.32, \text{ i.e. } W < 0.6H \]

\[ \therefore C = \frac{2}{k} \frac{(1 - \frac{W}{1.2H}) - \frac{M}{2}}{\frac{q}{\sigma} (x_b + 1) + 40} \frac{0}{H} = 6.0 \text{ m} \quad (\text{Equation 4.4.7}) \]

and \[ Q = 2 (C + x_b) = 19.6 \text{ m} \quad (\text{Equation 4.5.1}) \]

This value of \( Q \) lies between the limits found in practice.

The accuracy of the formulae proved at Riddings Drift encouraged the use of this type of method to fix the pillar sizes at the neighbouring North Gawber Colliery. Here the faces were won successfully and without difficulty, there being a substantial increase in percentage recovery over that previously proposed.

(e) **Pillar Size in the Beeston Seam at Kellingley Colliery**

A recent publication [51] outlines experience with pillar sizes in the Beeston Seam at Kellingley Colliery. This allows the hypothetically calculated pillar size to be compared with
the observed behaviour of the pillars.

Information supplied:
- depth of cover, $H = 650$ m ($q = 16.25$ MPa)
- actual face length 275 m, roadway width 5 m
- total width of extraction, $W = 285$ m
- roadway height, $M = 3.66$ m

Information assumed:
- $\sigma_0 = 4$ MPa
- $p + p' = 0.1$ MPa
- $k = 3.75$ (i.e., mean of 4.0 and 3.5)

\[ x_b = \frac{M}{2} \left\{ \frac{q}{p + p'} \right\}^{\frac{2}{k-1}} - 1 \Rightarrow 9.8 \text{ m} \quad (\text{Equation 4.3.7}) \]

\[ \frac{W}{H} = \frac{285}{650} = 0.44, \text{ i.e. } W < 0.6 \text{ H} \]

\[ \frac{W}{2} \left( 1 - \frac{W}{1.2H} \right) - \frac{M}{2} = 29.6 \text{ m} \quad (\text{Equation 4.4.7}) \]

\[ \sigma = k\sigma_0 = 65 \text{ MPa} \quad (\text{Equation 4.3.3}) \]

\[ Q = 2(C + x_b) = 79 \text{ m} \quad (\text{Equation 4.5.1}) \]

To quote from the publication (p 400) as to the actual behaviour of pillars in the Beeston Seam:

"working the Beeston Seam at a depth of about 650 metres has established empirical pillar dimensions which have influenced design:

45 metres - impossible to maintain the roadway"
60 metres - severe roadway distortion necessitating replacement of arches

80 metres - limited roadway distortion requiring dinting and occasional back ripping over 80 metres - dinting only required".

In Figure 30 the hypothetical exponential stress decay and the equivalent area triangular stress distribution have both been plotted, and the 45 m, 60 m and 80 m pillar widths indicated. The damage to the roadways quoted appears to give a somewhat better agreement with the exponential distribution, but the general concept of fixing the minimum pillar by $Q = 2 (C + x)$ is confirmed.

(f) Pillars in a Canadian Coal Mine

In a Canadian coal mine, experience has been gained of mining coal at depths of from 100 m to 700 m. During the course of this extraction, pillar sizes have been fixed using a formula

$$\frac{H}{1-R} = 7,000$$

where $H$ is the depth in feet and $R$ the fraction of extraction. By substituting $R = \frac{W}{Q + W}$, this equation can be transposed to

$$Q = \frac{HW}{7,000 - H}$$

where $Q =$ minimum pillar width (ft)

$H =$ depth of cover (ft)

$W =$ length of longwall face (ft)

This is a common type of formula used in North America, and is based on the assumption that there is an even stress across the pillar and that the area between the pillars takes no load.
Figure 30. Hypothetical Stress Decay in Beeston Seam
\((H = 650 \text{ m}, W = 285 \text{ m})\)
Under such conditions

pillar stress \times \text{pillar width} = \text{total area supported} \times \text{cover} \times \text{density, i.e.}

\[ cQ = (Q + W) \frac{H_y}{\gamma} \]

\[ \frac{H_y}{\gamma} = \frac{Q}{Q + W} = 1 - \frac{W}{Q + W} = 1 - R \]

\[ \therefore \frac{H}{1 - R} = \frac{c}{\gamma} = \text{a constant} \]

As mining became deeper, there was realisation on the part of the management that pillars calculated by this formula were probably wider than they needed to be. Moreover, mining to a depth of 1400 m was planned, and at such depth the minimum pillar size of 414 m (between 217 m longwall faces — the standard in the mine) was obviously far too wide.

The mine was visited and samples for laboratory testing collected from roof, seam and floor. The following data were determined:

laboratory strength:
roof \quad 48 \text{ MPa}
seam \quad 17 \text{ MPa}
floor \quad 67 \text{ MPa}

(i.e. the seam was weak compared to the roof and floor)

in situ strength of seam, \( \sigma = \frac{17}{5} = 3.4 \text{ MPa} \)
triaxial stress factor, \( k = 3.0 \)
face width, \( W = 217 \text{ m} \)
roadway height, \( M = 2.4 \text{ m} \)
degree of total support, \( p + p' = 0.3 \text{ MPa} \)
Using Equations 4.3.5, 4.4.4, 4.4.5 and 4.5.1, hypothetical minimum pillar sizes were calculated over a range of depth from 100 m to 1400 m. These calculations are plotted in Figure 31. For comparison, pillar sizes based on the formulae \( \frac{H \text{ (ft)}}{1 - R} = 7,000 \) and \( Q = 0.1 \times H + 15 \text{ yds} \) have also been indicated.

Over the depth range 100 m to 600 m (the range over which \( \frac{H \text{ (ft)}}{1 - R} = 7,000 \) had been confirmed), all three methods provide similar results. Thereafter they diverge, the hypothetical formulae giving the lower and probably more realistic values.

(g) Pillars in the Pittsburgh Coal Seam

At a mine in Pennsylvania, an estimate of the loading was required on pillars formed by a triple entry system after the extraction of longwall faces. The seam was located at a depth of 183 m, the roadways were approximately 3 m x 3 m, and two pillars of 13.4 m and of 21 m in width were formed by the three roadways. It was estimated that the total boundary resistance was about 0.05 MPa, the strength of the coal in situ 5 MPa, and the triaxial stress factor of the order of 4. Roof, floor and seam were comparable in strength, and the width of the faces exceeded 0.6 H.

It was assumed that the various stress increases produced by more than one excavation were additive, and the hypothetical equations were used to estimate the build-up of load on the pillars as mining proceeded.

(1) Stresses after initial roadway drivage:

stress field \( q = \gamma H = 4.58 \text{ MPa} \)
FIGURE 31. MINIMUM PILLAR WIDTH REQUIRED BETWEEN 217 m WIDE FACES IN A CANADIAN MINE (SOFT COAL BETWEEN STRONG ROOF AND FLOOR)
radius of yield boundary, \( r = r_0 \left( \frac{2q - c_o}{(p + p') (k + 1)} \right)^\frac{1}{k-1} \)

\( r_0 = \text{radius of opening} \)

\( = 3.8 \text{ m} \)  \hspace{1cm} (Equation 3.3.9)

abutment increase, \( \hat{c} - q = \frac{k (2q - c_o)}{k + 1} + c_o - q \)

\( = 3.7 \text{ MPa} \)  \hspace{1cm} (Equation 3.3.5)

elastic constant, \( A = \left( \frac{(k-1)q + c_o}{k + 1} \right)^{\frac{2}{k-1}} \cdot \left( \frac{2q - c}{(p + p') (k + 1)} \right)^{\frac{k}{k-1}} \)

\( = 24.4 \text{ MPa} \)  \hspace{1cm} (Equation 3.3.8)

stress increase in elastic zone

\( (\sigma - q) = A \left( r \right)^2 \cdot \frac{1.5}{r} \)

\( = 24.4 \left( \frac{1.5}{r} \right) \text{ MPa} \)  \hspace{1cm} (Equation 3.3.7)

This allowed Figure 32a to be constructed.

(ii) After extraction of the left-hand face:

width of yield zone, \( x = \frac{M}{2} \left( \frac{q}{p + p'} - 1 \right) \)

\( = 5.3 \text{ m} \)  \hspace{1cm} (Equation 4.3.7)

abutment increase, \( \hat{c} - q = kq + c_o - q \)

\( = 18.7 \text{ MPa} \)  \hspace{1cm} (Equation 4.3.3)

exponential decay factor, \( C = \frac{0.15 H - \frac{M}{2}}{(k-1) + 40 \frac{c_o}{H}} \)

\( = 6.34 \text{ m} \)  \hspace{1cm} (Equation 4.4.6)
FIGURE 32. STRESS DISTRIBUTION ACROSS PILLARS IN PITTSBURGH SEAM
stress increase in elastic zone, \((\tau - q)\)

\[
= (\tau - q) \cdot e^{\frac{x_0 - x}{c}}
\]

\[
= 18.7 \cdot e^{\frac{5.3 - x}{2.32}} \text{ MPa} \quad \text{(Equation 4.4.1)}
\]

The determination of these values permits the plotting of the abutment peak and stress decay (ignoring the effect of the middle roadway), as shown by the dotted line in Figure 32b.

The general stress in the neighbourhood of the middle roadway increases as a consequence of extracting the coal at the face. This increased stress must be redistributed. As an approximation, it is assumed that this can be done by substituting \(q'\) (see Figure 32b) for \(q\) in Equations 3.3.9, 3.3.5, 3.3.8 and 3.3.7 above. The resultant modified stress distribution is shown by the solid line in Figure 32b.

(iii) After extraction of both faces:

The same procedure was repeated for the second face. The final hypothetical stress distribution is shown in Figure 32c. It should be noted that, when superimposing the overlapping stress fields, only that portion of the stress above the original cover load is additive, the cover load stress itself only being added in once. As an illustration, the method of adding the stress increments for some of the salient points is shown in Table 4.

It must be emphasised that this method is only an approximation, pending a more rigorous solution. Basically, it amounts to a redistribution of the stress deficiencies created by the excavation of the faces and roadways in a logical manner, which follows the laws established in the earlier parts of this thesis.
<table>
<thead>
<tr>
<th>Distance from left-hand rib, (x) (m)</th>
<th>5.3</th>
<th>7.2</th>
<th>9.0</th>
<th>14.9 ((r=0))</th>
<th>20.8</th>
<th>24.6</th>
<th>28.3</th>
<th>32.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st face, (\Delta \sigma = 18.74 e^{6.34})</td>
<td>18.7</td>
<td>13.9</td>
<td>10.5</td>
<td>4.1</td>
<td>1.6</td>
<td>0.9</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>2nd face, (\Delta \sigma = 18.74 e^{6.34})</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>1.2</td>
<td>3.2</td>
<td>5.7</td>
<td>10.3</td>
<td>18.7</td>
</tr>
<tr>
<td>Middle road, (\Delta \sigma = 106 \left(\frac{1.5}{r}\right)^2)</td>
<td>2.6</td>
<td>4.0</td>
<td>6.9</td>
<td>-9.9</td>
<td>6.9</td>
<td>2.5</td>
<td>1.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Cover load = (\gamma H)</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
</tr>
<tr>
<td>TOTAL (MPa)</td>
<td>26.2</td>
<td>22.9</td>
<td>22.5</td>
<td>0</td>
<td>16.3</td>
<td>13.7</td>
<td>16.7</td>
<td>24.4</td>
</tr>
</tbody>
</table>
Concerning the interpretation of the stress diagrams shown in Figure 32, it is obvious that the initial roadway drivage will have little effect on the stability of the strata. The extraction of the first face will only slightly affect the right-hand roadway, but will significantly increase the width of the yield zone around the middle roadway, with resultant increase in closure. Extraction of the second face will add to the difficulties in the middle roadway.

Inspection of the pillars underground was possible while the faces were being extracted, and the interpretation was confirmed.

(a) **Pillar Reinforcement at Bagworth Colliery**

The shafts at Bagworth were originally sunk in 1903. In 1966 a trunk road (depth 133 m) was driven close to one of the shafts, as shown in Figure 33a. Movement of the shaft walls began, especially on the east side, and, despite repairs, deterioration of the shaft and inset walls continued. In the spring of 1971 the insets were heavily reinforced with steel joists at the approximate mid-height, but this reinforcement could not be continued across the shaft because of winding to a lower horizon. Closure of the insets was controlled, but the shaft walls continued to converge, as is shown in Figure 33c. A decision was taken to bolt the area of the shaft from the trunk road, the bolts passing right through the pillar, with bearing plates on each end. A determination of the density of bolting was required.
a. Plan of shaft, inset and roadway  
b. Stress diagram

c. Closure of shaft and inset (details supplied by colliery)

FIGURE 33. MINE INSET, BAGWORTH COLLIERY
A rock sample obtained from the pillar showed it to be a weak mudstone. It was too broken to obtain a core for determination of the unidirectional compressive strength, but, by fitting pieces together, an approximate value of 2.75 was obtained for the triaxial stress factor $k$.

If the stress rise in the yielding pillar is assumed to be of the form $\sigma = p k \left( \frac{R}{R_0} \right)^{k-1}$ (see Equation 3.3.2), then the stress diagram for the pillar will be as shown in Figure 33b. The load per unit length which such a pillar will accept is represented by the area $A$.

Using the nomenclature shown in Figure 33b,

$$\bar{\sigma} = p k \left( \frac{r}{r_1} \right)^{k-1} = p k \left( \frac{r_0 - \bar{r}}{r_2} \right)^{k-1}$$

$$\bar{r} = \frac{r_1}{r_1 + r_2} Q'$$

and

$$Q' = Q - \bar{r} = \frac{r_0 - \bar{r}}{r_1 + r_2} Q'$$

$$A = \int_{r_1}^{r_2} p k \left( \frac{r}{r_1} \right)^{k-1} \cdot dr + \int_{r_1}^{r_2} p k \left( \frac{r}{r_2} \right)^{k-1} \cdot dr$$

$$= p (r_1 + r_2) \left\{ \left( \frac{Q'}{r_1 + r_2} \right)^k - 1 \right\}$$

The load per unit length actually put on the pillar will probably be the cover load $\times$ (pillar width + half of each roadway) = $\gamma H \times Q'$.

Hence

$$\gamma H Q' = p (r_1 + r_2) \left\{ \left( \frac{Q'}{r_1 + r_2} \right)^k - 1 \right\}$$
Substitution of the requisite values gave \( p = 0.49 \, \text{MPa} \) for a value of \( k = 2.75 \). This latter value was only approximate. The pillar in situ was very wet, so the calculation was repeated using a value of \( k = 2.0 \). This gave \( p = 1.22 \, \text{MPa} \).

In the event, a density of bolting sufficient to give \( p = 2.25 \, \text{MPa} \) was used, the bolts being inserted during the latter part of the summer of 1972. The effect is shown clearly in Figure 33c and the shaft closure which has since occurred is attributed to the movement of the west wall only.

4.8 Conclusion

A set of formulae which allows the calculation of the minimum pillar width for roadway protection has been developed. Although to a large extent built up from concepts difficult to prove in practice, the overall deductions do fit the observed facts. The formulae have already been quite widely used in practice, but quoted examples have been limited to cases where some form of verification was possible.

For wide extractions on both sides of the pillar, the formulae give results similar to the 'one tenth depth plus 15 yd' rule. However, extractions less than the critical width can now be taken into account, and the role of support resistance can be better appreciated.

The triaxial stress factor (or its equivalent, the angle of internal friction) plays a dominant role in the calculation, a parameter hitherto ignored. The low value of triaxial stress factor in some seams, such as the Blackshale in Nottinghamshire and the Barnsley in Yorkshire, explains the difficult roadway conditions associated with these seams.